

# Package ‘DRPT’

July 7, 2025

**Title** Density Ratio Permutation Test

**Version** 1.1

**Description** Implementation of the Density Ratio Permutation Test for testing the goodness-of-fit of a hypothesised ratio of two densities, as described in Bordinino and Berrett (2025) <[doi:10.48550/arXiv.2505.24529](https://doi.org/10.48550/arXiv.2505.24529)>.

**License** MIT + file LICENSE

**Encoding** UTF-8

**RoxygenNote** 7.3.2

**LinkingTo** Rcpp

**Imports** Rcpp, BiasedUrn, rootSolve, future, future.apply, Rdpack (>= 2.6)

**RdMacros** Rdpack

**Suggests** testthat (>= 3.0.0)

**Config/testthat/edition** 3

**NeedsCompilation** yes

**Author** Alberto Bordinino [aut, cre] (ORCID: <<https://orcid.org/0009-0006-1556-6973>>),  
Thomas B. Berrett [aut] (ORCID: <<https://orcid.org/0000-0002-2005-110X>>)

**Maintainer** Alberto Bordinino <[alberto.bordinino@warwick.ac.uk](mailto:alberto.bordinino@warwick.ac.uk)>

**Repository** CRAN

**Date/Publication** 2025-07-07 09:10:05 UTC

## Contents

discrete.DRPT . . . . .	2
discreteT . . . . .	3
DRPT . . . . .	4
shiftedMMD . . . . .	5
starSampler . . . . .	6
<b>Index</b>	<b>8</b>

---

discrete.DRPT	<i>A function implementing the discrete version of the DRPT for discrete data with finite support.</i>
---------------	--------------------------------------------------------------------------------------------------------

---

### Description

A function that implements the discrete version of the DRPT for discrete data with finite support as defined in Section 2.1 in Bordino and Berrett (2025).

### Usage

```
discrete.DRPT(X, Y, r, H = 99, type = "V")
```

### Arguments

X	A numeric vector containing the first sample.
Y	A numeric vector containing the second sample.
r	A numeric vector of positive values specifying the hypothesised density ratio in the discrete setting.
H	An integer specifying the number of permutations to use. Defaults to 99.
type	A character string indicating the test statistic to use. See the Details section for more information. Defaults to "V".

### Details

Counts for the permuted samples are drawn using `rMFNCHypergeo` from the package `BiasedUrn`. When `type="U"` the test statistic is the U-statistic (12); when `type="V"` the test statistic is the V-statistic (11); setting `type="D"` gives the test statistic (56) in Appendix B of the paper.

### Value

The p-value of the DRPT as defined in (2) in Bordino and Berrett (2025).

### References

Bordino A, Berrett TB (2025). "Density Ratio Permutation Tests with connections to distributional shifts and conditional two-sample testing." arXiv:2505.24529, <https://arxiv.org/abs/2505.24529>.

### Examples

```
n = 100; m = n
X = sample(0:3, n, prob = c(1/8, 1/8, 3/8, 3/8), replace = TRUE)
Y = sample(0:3, m, prob = c(1/43, 3/43, 16/43, 23/43), replace = TRUE)
r = c(1, 3, 3, 10)

discrete.DRPT(X, Y, r, H=19)
```

```
discrete.DRPT(X,Y,r, type = "U", H=19)
discrete.DRPT(X,Y,r, type = "D", H=19)
```

---

discreteT

---

*Compute test statistics for the DRPT in discrete settings.*


---

## Description

Computes the test statistics introduced in Bordino and Berrett (2025) for settings where the data support is discrete and finite.

## Usage

```
discreteT(NX, NY, r, n, m, type = "V")
```

## Arguments

NX	A vector of counts for the first sample. This corresponds to the sequence $\text{tot}_j - N_{Y,j}^p$ with $p = \text{id}$ , i.e. the identity permutation, as introduced in Section 2.1 of Bordino and Berrett (2025).
NY	A vector of counts for the second sample. This corresponds to the sequence $N_{Y,j}^p$ with $p = \text{id}$ , i.e. the identity permutation, as introduced in Section 2.1 of Bordino and Berrett (2025).
r	A numeric vector of positive values specifying the hypothesised density ratio in the discrete setting.
n	The size of the first sample.
m	The size of the second sample.
type	A character string indicating which test statistic to compute. One of "U", "V", or "D". See the Details section for more information. Defaults to "V".

## Details

When `type = "U"`, the U-statistic (12) is calculated. When `type = "V"`, the V-statistic (11) is computed. When `type = "D"`, the test statistic (56) from Appendix B is returned.

## Value

A numeric value representing the computed test statistic.

## References

Bordino A, Berrett TB (2025). "Density Ratio Permutation Tests with connections to distributional shifts and conditional two-sample testing." arXiv:2505.24529, <https://arxiv.org/abs/2505.24529>.

**Examples**

```

n = 100; m = n
X = sample(0:3, n, prob = c(1/4, 1/4, 1/4, 1/4), replace = TRUE)
Y = sample(0:3, m, prob = c(1/17, 3/17, 3/17, 10/17), replace = TRUE)
r = c(1, 3, 3, 10)

NX = table(X)
NY = table(Y)

discreteT(NX, NY, r, sum(NX), sum(NY), type = "V")
discreteT(NX, NY, r, sum(NX), sum(NY), type = "D")

```

---

DRPT

*A function implementing the Density Ratio Permutation Test based on an estimate of the shifted-MMD.*

---

**Description**

A function that implements the DRPT based on the U-statistic (12) defined in Bordino and Berrett (2025). An estimator of the shifted-MMD with kernel  $k(\cdot, \cdot)$  as defined in Section 3.2 of the paper is computed using the function `shiftedMMD`, which is provided in the package.

**Usage**

```
DRPT(X, Y, r, kernel, H = 99, S = 50)
```

**Arguments**

X	A numeric vector containing the first sample.
Y	A numeric vector containing the second sample.
r	A function specifying the hypothesised density ratio.
kernel	A function defining the kernel to be used for the U-statistic.
H	An integer specifying the number of permutations to use. Defaults to 99.
S	An integer specifying the number of steps for the Markov-Chain defined in Algorithm 2 in Bordino and Berrett (2025). Defaults to 50.

**Value**

The p-value of the DRPT as defined in (2) in Bordino and Berrett (2025).

**References**

Bordino A, Berrett TB (2025). “Density Ratio Permutation Tests with connections to distributional shifts and conditional two-sample testing.” arXiv:2505.24529, <https://arxiv.org/abs/2505.24529>.

## Examples

```
n = 50; m = 50; d = 2
r = function(x,y) {
  return(4*x*y)
}

gaussian.kernel = function(x, y, lambda = 1){
  return(lambda^(-d) * exp(-sum(((x - y) ^ 2) / (lambda ^ 2))))
}

X = as.matrix(cbind(runif(n, 0, 1), runif(n, 0, 1)))
Y = as.matrix(cbind(rbeta(m, 0.5, 0.3), rbeta(m, 0.5, 0.4)))

DRPT(X,Y, r, gaussian.kernel, H=19, S=10)
DRPT(X,Y, r, gaussian.kernel, H=9)
```

---

shiftedMMD

*A function computing an estimate of the shifted-MMD.*

---

## Description

A function computing the U-statistic (12). This serves as an estimator of the shifted-MMD defined in Section 3.2 of Bordino and Berrett (2025).

## Usage

```
shiftedMMD(X, Y, r, kernel)
```

## Arguments

X	A numeric vector containing the first sample.
Y	A numeric vector containing the second sample.
r	A function specifying the hypothesised density ratio.
kernel	A function defining the kernel to be used for the U-statistic.

## Value

The value of the U-statistic (12).

## References

Bordino A, Berrett TB (2025). “Density Ratio Permutation Tests with connections to distributional shifts and conditional two-sample testing.” arXiv:2505.24529, <https://arxiv.org/abs/2505.24529>.

**Examples**

```

n = 250; m = 250; d = 2
r = function(x,y) {
  return(4*x*y)
}

gaussian.kernel = function(x, y, lambda = 1){
  return(lambda^(-d) * exp(-sum(((x - y) ^ 2) / (lambda ^ 2))))
}

X = as.matrix(cbind(runif(n, 0, 1), runif(n, 0, 1)))
Y = as.matrix(cbind(rbeta(m, 0.5, 0.3), rbeta(m, 0.5, 0.4)))

shiftedMMD(X,Y, r, gaussian.kernel)

```

---

starSampler

*A function implementing the star-sampler for the DRPT.*


---

**Description**

A function implementing Algorithm 2 in Bordino and Berrett (2025).

**Usage**

```
starSampler(X, Y, r, H = 99, S = 50)
```

**Arguments**

X	A numeric vector containing the first sample.
Y	A numeric vector containing the second sample.
r	A function specifying the hypothesised density ratio.
H	An integer specifying the number of permutations to use. Defaults to 99.
S	An integer specifying the number of steps for the Markov-Chain defined in Algorithm 2 in Bordino and Berrett (2025). Defaults to 50.

**Value**

A list of  $H + 1$  rearrangements of the whole sample. The first element of the list is the original dataset. The other  $H$  elements are permutations of the original dataset, where permutations are generated using Algorithm 2 in the paper.

**References**

Bordino A, Berrett TB (2025). “Density Ratio Permutation Tests with connections to distributional shifts and conditional two-sample testing.” arXiv:2505.24529, <https://arxiv.org/abs/2505.24529>.

**Examples**

```
n = 250; m = n
r = function(x,y) {
  return(4*x*y)
}

X = as.matrix(cbind(runif(n, 0, 1), runif(n, 0, 1)))
Y = as.matrix(cbind(rbeta(m, 0.5, 0.3), rbeta(m, 0.5, 0.4)))

starSampler(X, Y, r, H = 3, S = 20)
```

# Index

discrete.DRPT, [2](#)  
discreteT, [3](#)  
DRPT, [4](#)  
  
shiftedMMD, [5](#)  
starSampler, [6](#)