

# Package ‘MixtureFitting’

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**Title** Fitting of Univariate Mixture Distributions to Data using  
Various Approaches

**Depends** R (>= 2.0.1)

**Description** Methods for fitting mixture distributions to univariate data using expectation maximization, HWHM and other methods. Supports Gaussian, Cauchy, Student's t and von Mises mixtures. For more details see Merkys (2018) <[https://www.lvb.lt/permalink/370LABT\\_NETWORK/1m6ui06/alma9910036312108451](https://www.lvb.lt/permalink/370LABT_NETWORK/1m6ui06/alma9910036312108451)>.

**License** GPL-2

**NeedsCompilation** yes

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**Repository** CRAN

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|                              |                                    |
|------------------------------|------------------------------------|
| <code>abs_convergence</code> | <i>Absolute Convergence Check.</i> |
|------------------------------|------------------------------------|

---

**Description**

Compare two values to tell whether an optimization process has converged.

**Usage**

```
abs_convergence( p_now, p_prev, epsilon = 1e-6 )
```

**Arguments**

- `p_now`            function value of *i*-th iteration.
- `p_prev`          function value of *i-1*-th iteration.
- `epsilon`          convergence criterion

**Value**

TRUE if deemed to have converged, FALSE otherwise

**Author(s)**

Andrius Merkys

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|                                 |                                                                      |
|---------------------------------|----------------------------------------------------------------------|
| <code>bhattacharyya_dist</code> | <i>Bhattacharyya distance for univariate Gaussian distributions.</i> |
|---------------------------------|----------------------------------------------------------------------|

---

**Description**

Measures Bhattacharyya distance for two univariate Gaussian distributions.

**Usage**

```
bhattacharyya_dist( mu1, mu2, sigma1, sigma2 )
```

**Arguments**

- `mu1`            mean of the first Gaussian distribution.
- `mu2`            mean of the second Gaussian distribution.
- `sigma1`        standard deviation of the first Gaussian distribution.
- `sigma2`        standard deviation of the second Gaussian distribution.

**Value**

Bhattacharyya distance as double.

**Author(s)**

Andrius Merkys

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|     |                                             |
|-----|---------------------------------------------|
| bic | <i>Bayesian Information Criterion (BIC)</i> |
|-----|---------------------------------------------|

---

**Description**

Calculates Bayesian Information Criterion (BIC) for any type of mixture model. Log-likelihood function has to be provided.

**Usage**

```
bic( x, p, llf )
```

**Arguments**

|     |                                                            |
|-----|------------------------------------------------------------|
| x   | data vector                                                |
| p   | vector of mixture model parameters                         |
| llf | function calculating log-likelihood, called as llf( x, p ) |

**Value**

Bayesian Information Criterion value.

**Author(s)**

Andrius Merkys

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|            |                                                                           |
|------------|---------------------------------------------------------------------------|
| cmm_fit_em | <i>Estimate Cauchy Mixture parameters using Expectation Maximization.</i> |
|------------|---------------------------------------------------------------------------|

---

**Description**

Estimates parameters for Cauchy mixture using Expectation Maximization algorithm.

**Usage**

```
cmm_fit_em( x, p, epsilon = c( 0.000001, 0.000001, 0.000001 ),
            iter.cauchy = 20, debug = FALSE, implementation = "C" )
```

**Arguments**

|                |                                                                                                                                                                                                                                                                                                                                                    |
|----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x              | data vector                                                                                                                                                                                                                                                                                                                                        |
| p              | initialization vector of $3*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A1, A2, ..., An, mu1, mu2, ..., mun, gamma1, gamma2, ..., gamman )$ , where $A_i$ is the proportion of $i$ -th component, $mu_i$ is the center of $i$ -th component and $gamma_i$ is the Cauchy scale of $i$ -th component. |
| epsilon        | tolerance threshold for convergence. Structure of epsilon is $epsilon = c( epsilon\_A, epsilon\_mu, epsilon\_gamma )$ , where $epsilon\_A$ is threshold for component proportions, $epsilon\_mu$ is threshold for component centers and $epsilon\_gamma$ is threshold for component Cauchy scales.                                                 |
| iter.cauchy    | number of iterations to fit a single Cauchy component.                                                                                                                                                                                                                                                                                             |
| debug          | flag to turn the debug prints on/off.                                                                                                                                                                                                                                                                                                              |
| implementation | flag to switch between C (default) and R implementations.                                                                                                                                                                                                                                                                                          |

**Value**

Vector of mixture parameters, whose structure is the same as of input parameter's p.

**Author(s)**

Andrius Merkys

**References**

Ferenc Nahy. Parameter Estimation of the Cauchy Distribution in Information Theory Approach (2006). Journal of Universal Computer Science

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cmm\_fit\_hwhm\_spline\_deriv

*Estimate Cauchy Mixture Parameters Using Derivatives and Half-Width at Half-Maximum Method.*

---

**Description**

Estimate Cauchy mixture parameters using derivatives and half-width at half-maximum (HWHM) method. The method smooths the histogram before attempting to locate the modes. Then it describes them using HWHM.

**Usage**

```
cmm_fit_hwhm_spline_deriv( x, y )
```

**Arguments**

|   |                       |
|---|-----------------------|
| x | data vector           |
| y | response vector for x |

**Value**

Parameter vector of  $3*n$  parameters, where  $n$  is number of mixture components. Structure of  $p$  vector is  $p = c( A_1, A_2, \dots, A_n, \mu_1, \mu_2, \dots, \mu_n, \gamma_1, \gamma_2, \dots, \gamma_n )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the center of  $i$ -th component and  $\gamma_i$  is the Cauchy scale of  $i$ -th component.

**Author(s)**

Andrius Merkys

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|                 |                                                                           |
|-----------------|---------------------------------------------------------------------------|
| cmm_init_vector | <i>Estimate Cauchy Mixture parameters using Expectation Maximization.</i> |
|-----------------|---------------------------------------------------------------------------|

---

**Description**

Estimate an initialization vector for Cauchy mixture fitting via Expectation Maximization. Proportions are set to equal, centers are equispaced through the whole domain of input sample, and scales are set to 1.

**Usage**

```
cmm_init_vector( x, m, implementation = "C" )
```

**Arguments**

|                             |                                                           |
|-----------------------------|-----------------------------------------------------------|
| <code>x</code>              | data vector                                               |
| <code>m</code>              | number of mixture components                              |
| <code>implementation</code> | flag to switch between C (default) and R implementations. |

**Value**

Parameter vector of  $3*n$  parameters, where  $n$  is number of mixture components. Structure of  $p$  vector is  $p = c( A_1, A_2, \dots, A_n, \mu_1, \mu_2, \dots, \mu_n, \gamma_1, \gamma_2, \dots, \gamma_n )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the center of  $i$ -th component and  $\gamma_i$  is the Cauchy scale of  $i$ -th component.

**Author(s)**

Andrius Merkys

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`cmm_init_vector_kmeans`*Estimate Cauchy Mixture parameters using Expectation Maximization.*

---

## Description

Estimate an initialization vector for Cauchy mixture fitting using k-means. R implementation of k-means in `kmeans()` is used to find data point assignment to clusters. Then several iterations of Cauchy mixture fitting (per Nahy 2006) is used to derive mixture parameters.

## Usage

```
cmm_init_vector_kmeans( x, m, iter.cauchy = 20 )
```

## Arguments

|                          |                                                        |
|--------------------------|--------------------------------------------------------|
| <code>x</code>           | data vector                                            |
| <code>m</code>           | number of mixture components                           |
| <code>iter.cauchy</code> | number of iterations to fit a single Cauchy component. |

## Value

Parameter vector of  $3*n$  parameters, where  $n$  is number of mixture components. Structure of `p` vector is `p = c( A1, A2, ..., An, mu1, mu2, ..., mun, gamma1, gamma2, ..., gamman )`, where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the center of  $i$ -th component and  $\gamma_i$  is the Cauchy scale of  $i$ -th component.

## Author(s)

Andrius Merkys

## References

Ferenc Nahy. Parameter Estimation of the Cauchy Distribution in Information Theory Approach (2006). Journal of Universal Computer Science

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|                   |                                                  |
|-------------------|--------------------------------------------------|
| cmm_intersections | <i>Intersections of Two Cauchy Distributions</i> |
|-------------------|--------------------------------------------------|

---

**Description**

Finds intersections of two Cauchy distributions by finding roots of a quadratic equation.

**Usage**

```
cmm_intersections( p )
```

**Arguments**

**p** parameter vector of 6 parameters. Structure of p vector is  $p = c( A1, A2, \mu1, \mu2, \gamma1, \gamma2 )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the location of  $i$ -th component,  $\gamma_i$  is the Cauchy scale of  $i$ -th component.

**Value**

A vector of x values of intersections (zero, one or two). Returns NaN if both distributions are identical.

**Author(s)**

Andrius Merkys

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|       |                                                    |
|-------|----------------------------------------------------|
| dcgmm | <i>Density of The Cauchy-Gaussian Distribution</i> |
|-------|----------------------------------------------------|

---

**Description**

Density function for the Cauchy-Gaussian distribution, according to Eqn. 2 of Swami (2000).

**Usage**

```
dcgmm( x, p )
```

**Arguments**

**x** data vector

**p** parameter vector of  $5*n$  parameters, where  $n$  is number of mixture components. Structure of p vector is  $p = c( A1, A2, \dots, An, e1, e2, \dots, en, \mu1, \mu2, \dots, \mu n, \gamma1, \gamma2, \dots, \gamma n, \sigma1, \sigma2, \dots, \sigma n )$ , where  $A_i$  is the proportion of  $i$ -th component,  $e_i$  is the proportion of Cauchy subcomponent of  $i$ -th component,  $\mu_i$  is the center of  $i$ -th component,  $\gamma_i$  is the Cauchy concentration of  $i$ -th component and  $\sigma_i$  is the Gaussian standard deviation of  $i$ -th component.

**Value**

A vector.

**Author(s)**

Andrius Merkys

**References**

Swami, A. Non-Gaussian mixture models for detection and estimation in heavy-tailed noise 2000 IEEE International Conference on Acoustics, Speech, and Signal Processing. Proceedings (Cat. No.00CH37100), 2000, 6, 3802-3805

---

dcmm

*Density of The Cauchy Mixture Distribution*


---

**Description**

Density function for the Cauchy mixture distribution.

**Usage**

```
dcmm( x, p, implementation = "C" )
```

**Arguments**

**x** data vector

**p** parameter vector of  $3 \cdot n$  parameters, where  $n$  is number of mixture components. Structure of  $p$  vector is  $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, \gamma1, \gamma2, \dots, \gamma n )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the location of  $i$ -th component,  $\gamma_i$  is the Cauchy scale of  $i$ -th component.

**implementation** flag to switch between C (default) and R implementations.

**Value**

A vector.

**Author(s)**

Andrius Merkys

---

dgmm

*The Gaussian Mixture Distribution*


---

**Description**

Density function for the Gaussian mixture distribution.

**Usage**

```
dgmm( x, p, normalise_proportions = FALSE, restrict_sigmas = FALSE,
      implementation = "C" )
```

**Arguments**

**x** data vector

**p** parameter vector of  $3 \cdot n$  parameters, where  $n$  is number of mixture components. Structure of  $p$  vector is  $p = c( A_1, A_2, \dots, A_n, \mu_1, \mu_2, \dots, \mu_n, \sigma_1, \sigma_2, \dots, \sigma_n )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the location of  $i$ -th component,  $\sigma_i$  is the scale of  $i$ -th component.

**normalise\_proportions** if TRUE, make component proportions sum up to 1 by dividing each one of them by their sum (R implementation only).

**restrict\_sigmas** if TRUE, skip components with scales less or equal to zero (R implementation only).

**implementation** flag to switch between C (default) and R implementations.

**Value**

A vector.

**Author(s)**

Andrius Merkys

---

digamma\_approx

*Calculate Approximate Value of The Digamma Function.*


---

**Description**

Calculates approximate value of the digamma function using first eight non-zero members of asymptotic expression for digamma( $x$ ). Implemented according to Wikipedia.

**Usage**

```
digamma_approx( x )
```

**Arguments**

x                      data vector

**Value**

Digamma function value.

**Author(s)**

Andrius Merkys

**References**

Users of Wikipedia. Digamma function. [https://en.wikipedia.org/w/index.php?title=Digamma\\_function&oldid=708779689](https://en.wikipedia.org/w/index.php?title=Digamma_function&oldid=708779689)

---

ds

*Density of The Student's t Model*

---

**Description**

Density function for the Student's t Model. Wrapper around R's `dt()`, supporting center and concentration parameters.

**Usage**

```
ds( x, c, s, ni )
```

**Arguments**

x                      data vector  
c                      center  
s                      concentration  
ni                     degrees of freedom

**Value**

A vector.

**Author(s)**

Andrius Merkys

---

dsmm

*Density of The Student's t Mixture Model*


---

**Description**

Density function for the Student's t Mixture Model.

**Usage**

```
dsmm( x, p )
```

**Arguments**

**x** data vector

**p** parameter vector of  $4*n$  parameters, where  $n$  is number of mixture components. Structure of  $p$  vector is  $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, k1, k2, \dots, kn, ni1, ni2, \dots, nin )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the center of  $i$ -th component,  $k_i$  is the concentration of  $i$ -th component and  $ni_i$  is the degrees of freedom of  $i$ -th component.

**Value**

A vector.

**Author(s)**

Andrius Merkys

---

dvmm

*Density of The von Mises Mixture Model.*


---

**Description**

Density function for the von Mises Mixture Model.

**Usage**

```
dvmm( x, p, implementation = "C" )
```

**Arguments**

**x** data vector

**p** parameter vector of  $3*n$  parameters, where  $n$  is number of mixture components. Structure of  $p$  vector is  $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, k1, k2, \dots, kn )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the center of  $i$ -th component and  $k_i$  is the concentration of  $i$ -th component.

**implementation** flag to switch between C (default) and R implementations.

**Value**

A vector.

**Author(s)**

Andrius Merkys

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|            |                                                                             |
|------------|-----------------------------------------------------------------------------|
| gmm_fit_em | <i>Estimate Gaussian Mixture parameters using Expectation Maximization.</i> |
|------------|-----------------------------------------------------------------------------|

---

**Description**

Estimates parameters for Gaussian mixture using Expectation Maximization algorithm.

**Usage**

```
gmm_fit_em( x, p, w = numeric(), epsilon = c( 0.000001, 0.000001, 0.000001 ),
            debug = FALSE, implementation = "C", ... )
```

**Arguments**

|                |                                                                                                                                                                                                                                                                                                                                                             |
|----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x              | data vector                                                                                                                                                                                                                                                                                                                                                 |
| p              | initialization vector of $3*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, \sigma1, \sigma2, \dots, \sigma n )$ , where $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the center of $i$ -th component and $\sigma_i$ is the scale of $i$ -th component. |
| w              | weights of data points, must have the same length as the data vector; if not given or has different length, equal weights are assumed.                                                                                                                                                                                                                      |
| epsilon        | tolerance threshold for convergence. Structure of epsilon is $\epsilon = c( \epsilon_A, \epsilon_\mu, \epsilon_\sigma )$ , where $\epsilon_A$ is threshold for component proportions, $\epsilon_\mu$ is threshold for component centers and $\epsilon_\sigma$ is threshold for component scales.                                                            |
| debug          | flag to turn the debug prints on/off.                                                                                                                                                                                                                                                                                                                       |
| implementation | flag to switch between C (default) and R implementations.                                                                                                                                                                                                                                                                                                   |
| ...            | additional arguments passed to <code>gmm_fit_em_R()</code> when R implementation is used.                                                                                                                                                                                                                                                                   |

**Value**

Vector of mixture parameters, whose structure is the same as of input parameter's p.

**Author(s)**

Andrius Merkys

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|              |                                                                                      |
|--------------|--------------------------------------------------------------------------------------|
| gmm_fit_hwhm | <i>Estimate Gaussian Mixture Parameters Using Half-Width at Half-Maximum Method.</i> |
|--------------|--------------------------------------------------------------------------------------|

---

**Description**

Estimate Gaussian mixture parameters using half-width at half-maximum (HWHM) method. Given a histogram, the method attempts to locate most prominent modes and describe them using HWHM.

**Usage**

```
gmm_fit_hwhm( x, y, n )
```

**Arguments**

|   |                              |
|---|------------------------------|
| x | data vector                  |
| y | response vector for x        |
| n | number of mixture components |

**Value**

Parameter vector of  $3*n$  parameters, where  $n$  is number of mixture components. Structure of p vector is  $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, \sigma1, \sigma2, \dots, \sigma n )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the location of  $i$ -th component,  $\sigma_i$  is the scale of  $i$ -th component.

**Author(s)**

Andrius Merkys

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|                           |                                                                                                      |
|---------------------------|------------------------------------------------------------------------------------------------------|
| gmm_fit_hwhm_spline_deriv | <i>Estimate Gaussian Mixture Parameters Using Derivatives and Half-Width at Half-Maximum Method.</i> |
|---------------------------|------------------------------------------------------------------------------------------------------|

---

**Description**

Estimate Gaussian mixture parameters using derivatives and half-width at half-maximum (HWHM) method. The method smooths the histogram before attempting to locate the modes. Then it describes them using HWHM.

**Usage**

```
gmm_fit_hwhm_spline_deriv( x, y )
```

**Arguments**

|                |                                    |
|----------------|------------------------------------|
| <code>x</code> | data vector                        |
| <code>y</code> | response vector for <code>x</code> |

**Value**

Parameter vector of  $3*n$  parameters, where  $n$  is number of mixture components. Structure of `p` vector is `p = c( A1, A2, ..., An, mu1, mu2, ..., mun, sigma1, sigma2, ..., sigman )`, where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the location of  $i$ -th component,  $\sigma_i$  is the scale of  $i$ -th component.

**Author(s)**

Andrius Merkys

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|                             |                                                          |
|-----------------------------|----------------------------------------------------------|
| <code>gmm_fit_kmeans</code> | <i>Estimate Gaussian Mixture parameters from kmeans.</i> |
|-----------------------------|----------------------------------------------------------|

---

**Description**

Estimates parameters for Gaussian mixture using kmeans.

**Usage**

```
gmm_fit_kmeans( x, n )
```

**Arguments**

|                |                              |
|----------------|------------------------------|
| <code>x</code> | data vector                  |
| <code>n</code> | number of mixture components |

**Value**

Vector of  $3*n$  mixture parameters, where  $n$  is number of mixture components. Structure of `p` vector is `p = c( A1, A2, ..., An, mu1, mu2, ..., mun, sigma1, sigma2, ..., sigman )`, where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the location of  $i$ -th component,  $\sigma_i$  is the scale of  $i$ -th component.

**Author(s)**

Andrius Merkys

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|                 |                                                                             |
|-----------------|-----------------------------------------------------------------------------|
| gmm_init_vector | <i>Estimate Gaussian Mixture parameters using Expectation Maximization.</i> |
|-----------------|-----------------------------------------------------------------------------|

---

### Description

Estimate an initialization vector for Gaussian mixture fitting via Expectation Maximization. Proportions and scales are set to equal, centers are equispaced through the whole domain of input sample.

### Usage

```
gmm_init_vector( x, n, implementation = "C" )
```

### Arguments

|                |                                                           |
|----------------|-----------------------------------------------------------|
| x              | data vector                                               |
| n              | number of mixture components                              |
| implementation | flag to switch between C (default) and R implementations. |

### Value

Parameter vector of  $3*n$  parameters, where  $n$  is number of mixture components. Structure of p vector is  $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, \sigma1, \sigma2, \dots, \sigma n )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the location of  $i$ -th component,  $\sigma_i$  is the scale of  $i$ -th component.

### Author(s)

Andrius Merkys

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|                        |                                                                             |
|------------------------|-----------------------------------------------------------------------------|
| gmm_init_vector_kmeans | <i>Estimate Gaussian Mixture parameters using Expectation Maximization.</i> |
|------------------------|-----------------------------------------------------------------------------|

---

### Description

Estimate an initialization vector for Gaussian mixture fitting using k-means. R implementation of k-means in `kmeans()` is used to find data point assignment to clusters.

### Usage

```
gmm_init_vector_kmeans( x, m )
```

**Arguments**

|   |                              |
|---|------------------------------|
| x | data vector                  |
| m | number of mixture components |

**Value**

Parameter vector of  $3*n$  parameters, where  $n$  is number of mixture components. Structure of p vector is  $p = c( A_1, A_2, \dots, A_n, \mu_1, \mu_2, \dots, \mu_n, \sigma_1, \sigma_2, \dots, \sigma_n )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the location of  $i$ -th component,  $\sigma_i$  is the scale of  $i$ -th component.

**Author(s)**

Andrius Merkys

---

gmm\_init\_vector\_quantile

*Estimate Gaussian Mixture parameters using Expectation Maximization.*

---

**Description**

Estimate an initialization vector for Gaussian mixture fitting using (weighted) quantiles. Proportions and scales are set to equal, centers are placed at equispaced quantiles.

**Usage**

```
gmm_init_vector_quantile( x, m, w = numeric() )
```

**Arguments**

|   |                              |
|---|------------------------------|
| x | data vector                  |
| m | number of mixture components |
| w | weight vector                |

**Value**

Parameter vector of  $3*n$  parameters, where  $n$  is number of mixture components. Structure of p vector is  $p = c( A_1, A_2, \dots, A_n, \mu_1, \mu_2, \dots, \mu_n, \sigma_1, \sigma_2, \dots, \sigma_n )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the location of  $i$ -th component,  $\sigma_i$  is the scale of  $i$ -th component.

**Author(s)**

Andrius Merkys

---

|                   |                                                    |
|-------------------|----------------------------------------------------|
| gmm_intersections | <i>Intersections of Two Gaussian Distributions</i> |
|-------------------|----------------------------------------------------|

---

**Description**

Finds intersections of two Gaussian distributions by finding roots of a quadratic equation.

**Usage**

```
gmm_intersections( p )
```

**Arguments**

|   |                                                                                                                                                                                                                                                              |
|---|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| p | parameter vector of 6 parameters. Structure of p vector is $p = c( A1, A2, \mu1, \mu2, \sigma1, \sigma2 )$ , where $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the location of $i$ -th component, $\sigma_i$ is the scale of $i$ -th component. |
|---|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

**Value**

A vector of x values of intersections (zero, one or two). Returns NaN if both distributions are identical.

**Author(s)**

Andrius Merkys

---

|                      |                                                |
|----------------------|------------------------------------------------|
| gmm_merge_components | <i>Merge two Gaussian components into one.</i> |
|----------------------|------------------------------------------------|

---

**Description**

Merges  $i$ th and  $j$ th components of Gaussian mixture model. Implemented in the same venue as in [mergeparameters](#) of fpc.

**Usage**

```
gmm_merge_components( x, p, i, j )
```

**Arguments**

|   |                                                                                                                                                                                                                                                                                                                                   |
|---|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x | data vector                                                                                                                                                                                                                                                                                                                       |
| p | vector of Gaussian mixture parameters. Structure of p vector is $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, k1, k2, \dots, kn )$ , where $n$ is number of mixture components, $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the center of $i$ -th component, $k_i$ is the concentration of $i$ -th component. |
| i | index of the first component to be merged. Component with this index will be replaced by a merged one in the output.                                                                                                                                                                                                              |
| j | index of the second component to be merged. Component with this index will be removed in the output.                                                                                                                                                                                                                              |

**Value**

Vector of mixture parameters, whose structure is the same as of input parameter's p.

**Author(s)**

Andrius Merkys

**References**

Hennig, C. Methods for merging Gaussian mixture components *Advances in Data Analysis and Classification*, Springer Nature, 2010, 4, 3-34

---

`gmm_size_probability`    *The Gaussian Mixture Distribution*

---

**Description**

Calculates the posterior probability of a Gaussian mixture with  $n$  components. Internally, it attempts to maximize log-likelihood of data by calling `optim()` and returns the list as received from `optim()`.

**Usage**

```
gmm_size_probability( x, n, method = "SANN" )
```

**Arguments**

|        |                                                    |
|--------|----------------------------------------------------|
| x      | data vector                                        |
| n      | number of mixture components                       |
| method | optimization method passed to <code>optim()</code> |

**Value**

List representing the converged `optim()` run.

**Author(s)**

Andrius Merkys

---

gmm\_size\_probability\_nls*The Gaussian Mixture Distribution*

---

**Description**

Calculates the posterior probability of a Gaussian mixture with  $n$  components. Internally, it bins the data vector and calls `nls()` to optimize the mixture fit. Returns the list of the same form as received from `optim()`.

**Usage**

```
gmm_size_probability_nls( x, n, bins = 100, trace = FALSE )
```

**Arguments**

|                    |                                |
|--------------------|--------------------------------|
| <code>x</code>     | data vector                    |
| <code>n</code>     | number of mixture components   |
| <code>bins</code>  | number of bins                 |
| <code>trace</code> | should debug trace be printed? |

**Value**

List of the same form as received from `optim()`.

**Author(s)**

Andrius Merkys

---

gradient\_descent*Gradient Descent*

---

**Description**

Simple implementation of gradient descent method. Given a derivative function, it follows its decrease until convergence criterion is met.

**Usage**

```
gradient_descent( gradfn, start, gamma = 0.1, ..., epsilon = 0.01 )
```

**Arguments**

|         |                                                       |
|---------|-------------------------------------------------------|
| gradfn  | derivative function                                   |
| start   | starting value                                        |
| gamma   | learning rate                                         |
| ...     | additional arguments passed to derivative function    |
| epsilon | convergence threshold for absolute squared difference |

**Value**

log-likelihood

**Author(s)**

Andrius Merkys

---

|       |                                                                                                    |
|-------|----------------------------------------------------------------------------------------------------|
| kldiv | <i>Kullback–Leibler Divergence of <math>i</math>th Student's <math>t</math> Mixture component.</i> |
|-------|----------------------------------------------------------------------------------------------------|

---

**Description**

Measures Kullback–Leibler divergence of  $i$ th Student's  $t$  Mixture component using Dirac's delta function. Implemented according to Chen et al. (2004).

**Usage**

```
kldiv( x, p, k )
```

**Arguments**

|   |                                                                                                                                                                                                                                                                                                                                                                                                                       |
|---|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x | data vector                                                                                                                                                                                                                                                                                                                                                                                                           |
| p | vector of Student's $t$ mixture parameters. Structure of p vector is $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, k1, k2, \dots, kn, ni1, ni2, \dots, nin )$ , where $n$ is number of mixture components, $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the center of $i$ -th component, $k_i$ is the concentration of $i$ -th component and $n_i$ is the degrees of freedom of $i$ -th component. |
| k | number of the component.                                                                                                                                                                                                                                                                                                                                                                                              |

**Value**

Kullback–Leibler divergence as double.

**Author(s)**

Andrius Merkys

## References

Chen, S.; Wang, H. & Luo, B. Greedy EM Algorithm for Robust T-Mixture Modeling Third International Conference on Image and Graphics (ICIG'04), Institute of Electrical & Electronics Engineers (IEEE), 2004, 548–551

---

|                 |                                                |
|-----------------|------------------------------------------------|
| kmeans_circular | <i>K-Means Clustering for Points on Circle</i> |
|-----------------|------------------------------------------------|

---

## Description

Perform k-means clustering on angular data (in degrees).

## Usage

```
kmeans_circular( x, centers, iter.max = 10 )
```

## Arguments

|          |                                        |
|----------|----------------------------------------|
| x        | data vector                            |
| centers  | vector of initial centers (in degrees) |
| iter.max | maximum number of iterations           |

## Value

Vector of the same length as *centers* defining cluster centers (in degrees).

## Author(s)

Andrius Merkys

---

|       |                                          |
|-------|------------------------------------------|
| llcmm | <i>Log-likelihood for Cauchy Mixture</i> |
|-------|------------------------------------------|

---

## Description

Calculates log-likelihood for a given data vector using a Cauchy mixture distribution.

## Usage

```
llcmm( x, p, implementation = "C" )
```

**Arguments**

`x` data vector

`p` parameter vector of  $3*n$  parameters, where  $n$  is number of mixture components. Structure of `p` vector is `p = c( A1, A2, ..., An, mu1, mu2, ..., mun, gamma1, gamma2, ..., gamman )`, where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the center of  $i$ -th component and  $\gamma_i$  is the Cauchy scale of  $i$ -th component.

`implementation` flag to switch between C (default) and R implementations.

**Value**

log-likelihood

**Author(s)**

Andrius Merkys

---

llgmm

---

*Log-likelihood for Gaussian Mixture*


---

**Description**

Calculates log-likelihood for a given data vector using a Gaussian mixture distribution.

**Usage**

```
llgmm( x, p, implementation = "C" )
```

**Arguments**

`x` data vector

`p` parameter vector of  $3*n$  parameters, where  $n$  is number of mixture components. Structure of `p` vector is `p = c( A1, A2, ..., An, mu1, mu2, ..., mun, sigma1, sigma2, ..., sigman )`, where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the center of  $i$ -th component and  $\sigma_i$  is the scale of  $i$ -th component.

`implementation` flag to switch between C (default) and R implementations.

**Value**

log-likelihood

**Author(s)**

Andrius Merkys

---

|                    |                                            |
|--------------------|--------------------------------------------|
| llgmm_conservative | <i>Log-likelihood for Gaussian Mixture</i> |
|--------------------|--------------------------------------------|

---

**Description**

Calculates log-likelihood for a given data vector using a Gaussian mixture distribution. This is a straightforward implementation, different from llgmm() in that that it does not detect and shortcut edge cases.

**Usage**

```
llgmm_conservative( x, p )
```

**Arguments**

|   |                                                                                                                                                                                                                                                                                                                                                        |
|---|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x | data vector                                                                                                                                                                                                                                                                                                                                            |
| p | parameter vector of $3*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, \sigma1, \sigma2, \dots, \sigma n )$ , where $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the center of $i$ -th component and $\sigma_i$ is the scale of $i$ -th component. |

**Value**

log-likelihood

**Author(s)**

Andrius Merkys

---

|                |                                                     |
|----------------|-----------------------------------------------------|
| llgmm_opposite | <i>Opposite Log-likelihood for Gaussian Mixture</i> |
|----------------|-----------------------------------------------------|

---

**Description**

Calculates opposite log-likelihood for a given data vector using a Gaussian mixture distribution.

**Usage**

```
llgmm_opposite( x, p )
```

**Arguments**

|   |                                                                                                                                                                                                                                                                                                                                                        |
|---|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x | data vector                                                                                                                                                                                                                                                                                                                                            |
| p | parameter vector of $3*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, \sigma1, \sigma2, \dots, \sigma n )$ , where $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the center of $i$ -th component and $\sigma_i$ is the scale of $i$ -th component. |

**Value**

opposite log-likelihood (negated log-likelihood value)

**Author(s)**

Andrius Merkys

---

llsmm

*Log-likelihood for Student's t Mixture*

---

**Description**

Calculates log-likelihood for a given data vector using a Student's t mixture distribution.

**Usage**

```
llsmm( x, p )
```

**Arguments**

|   |                                                                                                                                                                                                                                                                                                                                                                                                                     |
|---|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x | data vector                                                                                                                                                                                                                                                                                                                                                                                                         |
| p | parameter vector of $4*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, k1, k2, \dots, kn, ni1, ni2, \dots, nin )$ , where $Ai$ is the proportion of $i$ -th component, $\mu i$ is the center of $i$ -th component, $ki$ is the concentration of $i$ -th component and $nii$ is the degrees of freedom of $i$ -th component. |

**Value**

log-likelihood

**Author(s)**

Andrius Merkys

---

|       |                                             |
|-------|---------------------------------------------|
| llvmm | <i>Log-likelihood for von Mises Mixture</i> |
|-------|---------------------------------------------|

---

**Description**

Calculates log-likelihood for a given data vector using a von Mises mixture distribution.

**Usage**

```
llvmm( x, p, implementation = "C" )
```

**Arguments**

|                |                                                                                                                                                                                                                                                                                                                                                   |
|----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x              | data vector                                                                                                                                                                                                                                                                                                                                       |
| p              | parameter vector of $3*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A_1, A_2, \dots, A_n, \mu_1, \mu_2, \dots, \mu_n, k_1, k_2, \dots, k_n )$ , where $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the center of $i$ -th component and $k_i$ is the concentration of $i$ -th component. |
| implementation | flag to switch between C (default) and R implementations.                                                                                                                                                                                                                                                                                         |

**Value**

log-likelihood

**Author(s)**

Andrius Merkys

---

|                |                                                      |
|----------------|------------------------------------------------------|
| llvmm_opposite | <i>Opposite Log-likelihood for von Mises Mixture</i> |
|----------------|------------------------------------------------------|

---

**Description**

Calculates opposite log-likelihood for a given data vector using a von Mises mixture distribution.

**Usage**

```
llvmm_opposite( x, p )
```

**Arguments**

|   |                                                                                                                                                                                                                                                                                                                                                   |
|---|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x | data vector                                                                                                                                                                                                                                                                                                                                       |
| p | parameter vector of $3*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A_1, A_2, \dots, A_n, \mu_1, \mu_2, \dots, \mu_n, k_1, k_2, \dots, k_n )$ , where $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the center of $i$ -th component and $k_i$ is the concentration of $i$ -th component. |

**Value**

opposite log-likelihood (negated log-likelihood value)

**Author(s)**

Andrius Merkys

---

mk\_fit\_images

*Mixture Distribution Modeling*

---

**Description**

Draw a PNG histogram with a mixture density on top of it for each iteration of mixture optimization process.

**Usage**

```
mk_fit_images( h, l, prefix = "img_" )
```

**Arguments**

|        |                                           |
|--------|-------------------------------------------|
| h      | histogram object, as returned from hist() |
| l      | list containing model vectors             |
| prefix | prefix of file name to write              |

**Author(s)**

Andrius Merkys

---

plot\_circular\_hist

*Mixture Distribution Modeling*

---

**Description**

Plot a circular histogram.

**Usage**

```
plot_circular_hist( x, breaks = 72, ball = 0.5, ... )
```

**Arguments**

|        |                               |
|--------|-------------------------------|
| x      | data vector                   |
| breaks | number of breaks in histogram |
| ball   | radius of the drawn circle    |
| ...    | parameters passed to plot()   |

**Author(s)**

Andrius Merkys

---

plot\_density*Mixture Distribution Modeling*

---

**Description**

Draw a PNG histogram with a mixture density on top of it.

**Usage**

```
plot_density( x, model, density_f, width, height,  
              cuts = 400, main = "",  
              filename = NULL,  
              obs_good = c(), obs_bad = c(),  
              scale_density = FALSE )
```

**Arguments**

|               |                                                    |
|---------------|----------------------------------------------------|
| x             | data vector                                        |
| cuts          | number of breaks in histogram                      |
| main          | main title of the plot                             |
| model         | model passed to density_f()                        |
| density_f     | probability density function                       |
| filename      | name of the file to write                          |
| width         | image width, passed to png()                       |
| height        | image height, passed to png()                      |
| obs_good      | vector of values to mark with rug() in green color |
| obs_bad       | vector of values to mark with rug() in red color   |
| scale_density | should probability density be scaled?              |

**Author(s)**

Andrius Merkys

---

|             |                                                                   |
|-------------|-------------------------------------------------------------------|
| polyroot_NR | <i>Find one real polynomial root using Newton–Raphson method.</i> |
|-------------|-------------------------------------------------------------------|

---

**Description**

Finds one real polynomial root using Newton–Raphson method, implemented according to Wikipedia.

**Usage**

```
polyroot_NR( p, init = 0, epsilon = 1e-6, debug = FALSE, implementation = "C" )
```

**Arguments**

|                |                                                           |
|----------------|-----------------------------------------------------------|
| p              | vector of polynomial coefficients.                        |
| init           | initial value.                                            |
| epsilon        | tolerance threshold for convergence.                      |
| debug          | flag to turn the debug prints on/off.                     |
| implementation | flag to switch between C (default) and R implementations. |

**Value**

Real polynomial root.

**Author(s)**

Andrius Merkys

**References**

Users of Wikipedia. Newton's method. [https://en.wikipedia.org/w/index.php?title=Newton%27s\\_method&oldid=710342140](https://en.wikipedia.org/w/index.php?title=Newton%27s_method&oldid=710342140)

---

|      |                                                                                 |
|------|---------------------------------------------------------------------------------|
| pssd | <i>Penalized Sum of Squared Differences Using Gaussian Mixture Distribution</i> |
|------|---------------------------------------------------------------------------------|

---

**Description**

Given two vectors of same length and a Gaussian mixture, calculate the penalized sum of squared differences (SSD) between the first vector and Gaussian mixture densities measured at points from second vector. Penalties are included for proportions and scales that are less than or equal to 0.

**Usage**

```
pssd( x, y, p )
```

Arguments

- x data vector
- y response vector
- p parameter vector of  $3 \cdot n$  parameters, where  $n$  is number of mixture components. Structure of  $p$  vector is  $p = c( A_1, A_2, \dots, A_n, \mu_1, \mu_2, \dots, \mu_n, \sigma_1, \sigma_2, \dots, \sigma_n )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the location of  $i$ -th component,  $\sigma_i$  is the scale of  $i$ -th component.

Value

Penalized sum of squared differences.

Author(s)

Andrius Merkys

---

|               |                                                                                 |
|---------------|---------------------------------------------------------------------------------|
| pssd_gradient | <i>Penalized Sum of Squared Differences Using Gaussian Mixture Distribution</i> |
|---------------|---------------------------------------------------------------------------------|

---

Description

Gradient (derivative) function of pssd().

Usage

```
pssd_gradient( x, y, p )
```

Arguments

- x data vector
- y response vector
- p parameter vector of  $3 \cdot n$  parameters, where  $n$  is number of mixture components. Structure of  $p$  vector is  $p = c( A_1, A_2, \dots, A_n, \mu_1, \mu_2, \dots, \mu_n, \sigma_1, \sigma_2, \dots, \sigma_n )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the location of  $i$ -th component,  $\sigma_i$  is the scale of  $i$ -th component.

Value

Gradient values measured at  $x$ .

Author(s)

Andrius Merkys

---

|                   |                                 |
|-------------------|---------------------------------|
| ratio_convergence | <i>Ratio Convergence Check.</i> |
|-------------------|---------------------------------|

---

**Description**

Compare two values to tell whether an optimization process has converged. The absolute difference between values of two iterations is divided by the value of previous iteration and compared to the epsilon value.

**Usage**

```
ratio_convergence( p_now, p_prev, epsilon = 1e-6 )
```

**Arguments**

|         |                                        |
|---------|----------------------------------------|
| p_now   | function value of $i$ -th iteration.   |
| p_prev  | function value of $i-1$ -th iteration. |
| epsilon | convergence criterion                  |

**Value**

TRUE if deemed to have converged, FALSE otherwise

**Author(s)**

Andrius Merkys

---

|      |                                                         |
|------|---------------------------------------------------------|
| rcmm | <i>Random Sample of The Cauchy Mixture Distribution</i> |
|------|---------------------------------------------------------|

---

**Description**

Generates a random sample of the Cauchy mixture distribution.

**Usage**

```
rcmm( n, p )
```

**Arguments**

|   |                                                                                                                                                                                                                                                                                                                                                                     |
|---|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| n | sample size                                                                                                                                                                                                                                                                                                                                                         |
| p | parameter vector of $3*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A_1, A_2, \dots, A_n, \mu_1, \mu_2, \dots, \mu_n, \gamma_1, \gamma_2, \dots, \gamma_n )$ , where $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the location of $i$ -th component, $\gamma_i$ is the Cauchy scale of $i$ -th component. |

**Value**

A vector.

**Author(s)**

Andrius Merkys

---

rgmm

*Random Sample of the Gaussian Mixture Distribution*

---

**Description**

Generates a random sample of the Gaussian mixture distribution.

**Usage**

```
rgmm( n, p )
```

**Arguments**

|   |                                                                                                                                                                                                                                                                                                                                                      |
|---|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| n | data vector                                                                                                                                                                                                                                                                                                                                          |
| p | parameter vector of $3*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, \sigma1, \sigma2, \dots, \sigma n )$ , where $Ai$ is the proportion of $i$ -th component, $\mu i$ is the location of $i$ -th component, $\sigma i$ is the scale of $i$ -th component. |

**Value**

A vector.

**Author(s)**

Andrius Merkys

---

|                |                                                                |
|----------------|----------------------------------------------------------------|
| rsimplex_start | <i>Nelder-Mead's Simplex Method for Function Minimization.</i> |
|----------------|----------------------------------------------------------------|

---

**Description**

Generate initial simplices for simplex().

**Usage**

```
rsimplex_start( seed, n, lower, upper )
```

**Arguments**

|       |                                            |
|-------|--------------------------------------------|
| seed  | seed for random number generator           |
| n     | number of simplices                        |
| lower | vector with lower bounds of each dimension |
| upper | vector with upper bounds of each dimension |

**Value**

A list with  $n$  simplices.

**Author(s)**

Andrius Merkys

---

|      |                                                      |
|------|------------------------------------------------------|
| rvmm | <i>Random Sample of the von Mises Mixture Model.</i> |
|------|------------------------------------------------------|

---

**Description**

Generates a random sample of the von Mises Mixture Model.

**Usage**

```
rvmm( n, p )
```

**Arguments**

|   |                                                                                                                                                                                                                                                                                                                                                         |
|---|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| n | sample size                                                                                                                                                                                                                                                                                                                                             |
| p | parameter vector of $3 \cdot n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A_1, A_2, \dots, A_n, \mu_1, \mu_2, \dots, \mu_n, k_1, k_2, \dots, k_n )$ , where $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the center of $i$ -th component and $k_i$ is the concentration of $i$ -th component. |

**Value**

A vector.

**Author(s)**

Andrius Merkys

**References**

Best & Fisher. Efficient Simulation of the von Mises Distribution. Journal of the RSS, Series C, 1979, 28, 152-157.

---

simplex

*Nelder-Mead's Simplex Method for Function Minimization.*

---

**Description**

Nelder-Mead's Simplex Method for Function Minimization.

**Usage**

```
simplex( fn, start, ..., epsilon = 0.000001, alpha = 1,
        gamma = 2, rho = 0.5, delta = 0.5, trace = FALSE )
```

**Arguments**

|         |                                                                        |
|---------|------------------------------------------------------------------------|
| fn      | minimized function, has to accept the argmin vector as first parameter |
| start   | start vector                                                           |
| ...     | other parameters passed to the minimized function                      |
| epsilon | convergence criterion                                                  |
| alpha   | reflection coefficient                                                 |
| gamma   | expansion coefficient                                                  |
| rho     | contraction coefficient                                                |
| delta   | shrink coefficient                                                     |
| trace   | should debug trace be printed?                                         |

**Value**

Vector yielding the minimum value of the minimized function

**Author(s)**

Andrius Merkys

## References

Nelder, J. A. & Mead, R. A Simplex Method For Function Minimization. The Computer Journal, 1965, 308-313.

Users of Wikipedia. Nelder-Mead method. [https://en.wikipedia.org/w/index.php?title=Nelder%E2%80%93Mead\\_method&oldid=1287347131](https://en.wikipedia.org/w/index.php?title=Nelder%E2%80%93Mead_method&oldid=1287347131)

---

|            |                                                                                |
|------------|--------------------------------------------------------------------------------|
| smm_fit_em | <i>Estimate Student's t Mixture parameters using Expectation Maximization.</i> |
|------------|--------------------------------------------------------------------------------|

---

## Description

Estimates parameters for Student's t mixture using Expectation Maximization algorithm. Calls smm\_fit\_em\_APK10().

## Usage

```
smm_fit_em( x, p, ... )
```

## Arguments

|     |                                                                                                                                                                                                                                                                                                                                                                                                                          |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x   | data vector                                                                                                                                                                                                                                                                                                                                                                                                              |
| p   | initialization vector of $4*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, k1, k2, \dots, kn, ni1, ni2, \dots, nin )$ , where $Ai$ is the proportion of $i$ -th component, $\mu i$ is the center of $i$ -th component, $ki$ is the concentration of $i$ -th component and $nii$ is the degrees of freedom of $i$ -th component. |
| ... | additional arguments passed to smm_fit_em_GNL08().                                                                                                                                                                                                                                                                                                                                                                       |

## Value

Vector of mixture parameters, whose structure is the same as of input parameter's p.

## Author(s)

Andrius Merkys

---

|                  |                                                                                |
|------------------|--------------------------------------------------------------------------------|
| smm_fit_em_APK10 | <i>Estimate Student's t Mixture parameters using Expectation Maximization.</i> |
|------------------|--------------------------------------------------------------------------------|

---

### Description

Estimates parameters for univariate Student's t mixture using Expectation Maximization algorithm, according to Fig. 2 of Aeschliman et al. (2010).

### Usage

```
smm_fit_em_APK10( x, p, epsilon = c( 1e-6, 1e-6, 1e-6, 1e-6 ),
                  collect.history = FALSE, debug = FALSE )
```

### Arguments

|                 |                                                                                                                                                                                                                                                                                                                                                                                                                             |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x               | data vector                                                                                                                                                                                                                                                                                                                                                                                                                 |
| p               | initialization vector of $4*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, k1, k2, \dots, kn, ni1, ni2, \dots, nin )$ , where $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the center of $i$ -th component, $k_i$ is the concentration of $i$ -th component and $ni_i$ is the degrees of freedom of $i$ -th component. |
| epsilon         | tolerance threshold for convergence. Structure of epsilon is $\epsilon = c( \epsilon_{A}, \epsilon_{\mu}, \epsilon_k, \epsilon_{ni} )$ , where $\epsilon_A$ is threshold for component proportions, $\epsilon_{\mu}$ is threshold for component centers, $\epsilon_k$ is threshold for component concentrations and $\epsilon_{ni}$ is threshold for component degrees of freedom.                                          |
| collect.history | flag to turn accumulation of estimation history on/off.                                                                                                                                                                                                                                                                                                                                                                     |
| debug           | flag to turn the debug prints on/off.                                                                                                                                                                                                                                                                                                                                                                                       |

### Value

A list.

### Author(s)

Andrius Merkys

### References

Aeschliman, C.; Park, J. & Kak, A. C. A Novel Parameter Estimation Algorithm for the Multivariate t-Distribution and Its Application to Computer Vision European Conference on Computer Vision 2010, 2010 <https://engineering.purdue.edu/RVL/Publications/Aeschliman2010ANovel.pdf>

---

smm\_fit\_em\_CWL04

*Greedy estimate Student's t Mixture parameters using Expectation Maximization.*

---

## Description

Estimates (greedily) parameters for univariate Student's t mixture using Expectation Maximization algorithm, implemented according to Chen et al. (2004). The algorithm relies upon `smm_fit_em_GNL08()` to estimate mixture parameters iteratively.

## Usage

```
smm_fit_em_CWL04( x, p, collect.history = FALSE, debug = FALSE,
  ... )
```

## Arguments

|                              |                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>               | data vector                                                                                                                                                                                                                                                                                                                                                                                                                            |
| <code>p</code>               | initialization vector of $4*n$ parameters, where $n$ is number of mixture components. Structure of <code>p</code> vector is <code>p = c( A1, A2, ..., An, mu1, mu2, ..., mun, k1, k2, ..., kn, ni1, ni2, ..., nin )</code> , where $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the center of $i$ -th component, $k_i$ is the concentration of $i$ -th component and $n_i$ is the degrees of freedom of $i$ -th component. |
| <code>collect.history</code> | logical. If set to TRUE, a list of parameter values of all iterations is returned.                                                                                                                                                                                                                                                                                                                                                     |
| <code>debug</code>           | flag to turn the debug prints on/off.                                                                                                                                                                                                                                                                                                                                                                                                  |
| <code>...</code>             | parameters passed to <code>smm_fit_em_GNL08()</code> .                                                                                                                                                                                                                                                                                                                                                                                 |

## Value

A list.

## Author(s)

Andrius Merkys

## References

Chen, S.; Wang, H. & Luo, B. Greedy EM Algorithm for Robust T-Mixture Modeling Third International Conference on Image and Graphics (ICIG'04), Institute of Electrical & Electronics Engineers (IEEE), 2004, 548–551

smm\_fit\_em\_GNL08

*Estimate Student's t Mixture parameters using Expectation Maximization.***Description**

Estimates parameters for univariate Student's t mixture using Expectation Maximization algorithm, according to Eqns. 12–17 of Gerogiannis et al. (2009).

**Usage**

```
smm_fit_em_GNL08( x, p, epsilon = c( 1e-6, 1e-6, 1e-6, 1e-6 ),
  collect.history = FALSE, debug = FALSE,
  min.sigma = 1e-256, min.ni = 1e-256,
  max.df = 1000, max.steps = Inf,
  polyroot.solution = 'jenkins_taub',
  convergence = abs_convergence,
  unif.component = FALSE )
```

**Arguments**

|                   |                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|-------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x                 | data vector                                                                                                                                                                                                                                                                                                                                                                                                                            |
| p                 | initialization vector of $4*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A_1, A_2, \dots, A_n, \mu_1, \mu_2, \dots, \mu_n, k_1, k_2, \dots, k_n, ni_1, ni_2, \dots, ni_n )$ , where $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the center of $i$ -th component, $k_i$ is the concentration of $i$ -th component and $ni_i$ is the degrees of freedom of $i$ -th component. |
| epsilon           | tolerance threshold for convergence. Structure of epsilon is $\epsilon = c( \epsilon_A, \epsilon_\mu, \epsilon_k, \epsilon_{ni} )$ , where $\epsilon_A$ is threshold for component proportions, $\epsilon_\mu$ is threshold for component centers, $\epsilon_k$ is threshold for component concentrations and $\epsilon_{ni}$ is threshold for component degrees of freedom.                                                           |
| collect.history   | logical. If set to TRUE, a list of parameter values of all iterations is returned.                                                                                                                                                                                                                                                                                                                                                     |
| debug             | flag to turn the debug prints on/off.                                                                                                                                                                                                                                                                                                                                                                                                  |
| min.sigma         | minimum value of sigma                                                                                                                                                                                                                                                                                                                                                                                                                 |
| min.ni            | minimum value of degrees of freedom                                                                                                                                                                                                                                                                                                                                                                                                    |
| max.df            | maximum value of degrees of freedom                                                                                                                                                                                                                                                                                                                                                                                                    |
| max.steps         | maximum number of steps, may be infinity                                                                                                                                                                                                                                                                                                                                                                                               |
| polyroot.solution | polyroot finding method used to approximate digamma function. Possible values are 'jenkins_taub' and 'newton_raphson'.                                                                                                                                                                                                                                                                                                                 |
| convergence       | function to use for convergence checking. Must accept function values of the last two iterations and return TRUE or FALSE.                                                                                                                                                                                                                                                                                                             |
| unif.component    | should a uniform component for outliers be added, as suggested by Cousineau & Chartier (2010)?                                                                                                                                                                                                                                                                                                                                         |

## Value

A list.

## Author(s)

Andrius Merkys

## References

Gerogiannis, D.; Nikou, C. & Likas, A. The mixtures of Student's t-distributions as a robust framework for rigid registration. Image and Vision Computing, Elsevier BV, 2009, 27, 1285–1294 <https://www.cs.uoi.gr/~arly/papers/imavis09.pdf>

Cousineau, D. & Chartier, S. Outliers detection and treatment: a review. International Journal of Psychological Research, 2010, 3, 58–67 <https://revistas.usb.edu.co/index.php/IJPR/article/view/844>

---

|                 |                                                                                |
|-----------------|--------------------------------------------------------------------------------|
| smm_init_vector | <i>Estimate Student's t Mixture parameters using Expectation Maximization.</i> |
|-----------------|--------------------------------------------------------------------------------|

---

## Description

Estimate an initialization vector for Student's t mixture fitting via Expectation Maximization. Proportions are set to be equal, centers are equispaced through the whole domain of input sample, concentrations and degrees of freedom are set to 1.

## Usage

```
smm_init_vector( x, n )
```

## Arguments

|   |                              |
|---|------------------------------|
| x | data vector                  |
| n | number of mixture components |

## Value

Parameter vector of  $4*n$  parameters, where  $n$  is number of mixture components. Structure of p vector is  $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, k1, k2, \dots, kn, ni1, ni2, \dots, nin )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the center of  $i$ -th component,  $k_i$  is the concentration of  $i$ -th component and  $ni_i$  is the degrees of freedom of  $i$ -th component.

## Author(s)

Andrius Merkys

---

smm\_init\_vector\_kmeans

*Estimate Student's t Mixture parameters using Expectation Maximization.*


---

### Description

Estimate an initialization vector for Student's t mixture fitting via Expectation Maximization. R implementation of k-means in kmeans() is used to find data point assignment to clusters. s\_fit\_primitive() is then used to estimate component parameters for each cluster.

### Usage

```
smm_init_vector_kmeans( x, m )
```

### Arguments

|   |                              |
|---|------------------------------|
| x | data vector                  |
| m | number of mixture components |

### Value

Parameter vector of  $3*n$  parameters, where  $n$  is number of mixture components. Structure of p vector is  $p = c( A_1, A_2, \dots, A_n, \mu_1, \mu_2, \dots, \mu_n, \sigma_1, \sigma_2, \dots, \sigma_n )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the location of  $i$ -th component,  $\sigma_i$  is the scale of  $i$ -th component.

### Author(s)

Andrius Merkys

---

smm\_split\_component     *Split a component of Student's t-distribution in two.*


---

### Description

Splits a component of Student's t-distribution mixture. Implemented according to Eqns. 30–36 of Chen et al. (2004).

### Usage

```
smm_split_component( p, alpha = 0.5, beta = 0.5, u = 0.5 )
```

### Arguments

|       |                                                                                                                                                                                                                                                                                                                                                                                                                      |
|-------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| p     | vector of Student's t mixture parameters. Structure of p vector is $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, k1, k2, \dots, kn, ni1, ni2, \dots, nin )$ , where $n$ is number of mixture components, $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the center of $i$ -th component, $k_i$ is the concentration of $i$ -th component and $ni_i$ is the degrees of freedom of $i$ -th component. |
| alpha | split proportion for component proportions                                                                                                                                                                                                                                                                                                                                                                           |
| beta  | split proportion for component concentrations                                                                                                                                                                                                                                                                                                                                                                        |
| u     | split proportion for component centers                                                                                                                                                                                                                                                                                                                                                                               |

### Value

Vector of parameters for resulting two-component mixture, whose structure is the same as of input parameter's p.

### Author(s)

Andrius Merkys

### References

Chen, S.-B. & Luo, B. Robust t-mixture modelling with SMEM algorithm Proceedings of 2004 International Conference on Machine Learning and Cybernetics (IEEE Cat. No.04EX826), Institute of Electrical & Electronics Engineers (IEEE), 2004, 6, 3689–3694

---

|     |                                                                       |
|-----|-----------------------------------------------------------------------|
| ssd | <i>Sum of Squared Differences Using Gaussian Mixture Distribution</i> |
|-----|-----------------------------------------------------------------------|

---

### Description

Given two vectors of same length and a Gaussian mixture, calculate the sum of squared differences (SSD) between the first vector and Gaussian mixture densities measured at points from second vector.

### Usage

```
ssd( x, y, p )
```

### Arguments

|   |                                                                                                                                                                                                                                                                                                                                                       |
|---|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x | data vector                                                                                                                                                                                                                                                                                                                                           |
| y | response vector                                                                                                                                                                                                                                                                                                                                       |
| p | parameter vector of $3*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, \sigma1, \sigma2, \dots, \sigma n )$ , where $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the location of $i$ -th component, $\sigma_i$ is the scale of $i$ -th component. |

**Value**

Sum of squared differences.

**Author(s)**

Andrius Merkys

---

ssd\_gradient

*Sum of Squared Differences Using Gaussian Mixture Distribution*

---

**Description**

Gradient (derivative) function of ssd().

**Usage**

```
ssd_gradient( x, y, p )
```

**Arguments**

|   |                                                                                                                                                                                                                                                                                                                                                       |
|---|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x | data vector                                                                                                                                                                                                                                                                                                                                           |
| y | response vector                                                                                                                                                                                                                                                                                                                                       |
| p | parameter vector of $3*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A1, A2, \dots, An, \mu1, \mu2, \dots, \mu n, \sigma1, \sigma2, \dots, \sigma n )$ , where $A_i$ is the proportion of $i$ -th component, $\mu_i$ is the location of $i$ -th component, $\sigma_i$ is the scale of $i$ -th component. |

**Value**

Gradient values measured at  $x$ .

**Author(s)**

Andrius Merkys

---

|                 |                                                                                          |
|-----------------|------------------------------------------------------------------------------------------|
| s_fit_primitive | <i>Estimate Student's t distribution parameters using Batch Approximation Algorithm.</i> |
|-----------------|------------------------------------------------------------------------------------------|

---

**Description**

Estimates parameters for univariate Student's t distribution parameters using Batch Approximation Algorithm, according to Fig. 2 of Aeschliman et al. (2010).

**Usage**

```
s_fit_primitive( x )
```

**Arguments**

|   |             |
|---|-------------|
| x | data vector |
|---|-------------|

**Value**

Vector  $c(\mu, k, n_i)$ , where  $\mu$  is the center,  $k$  is the concentration and  $n_i$  is the degrees of freedom of the distribution.

**Author(s)**

Andrius Merkys

**References**

Aeschliman, C.; Park, J. & Kak, A. C. A Novel Parameter Estimation Algorithm for the Multivariate t-Distribution and Its Application to Computer Vision European Conference on Computer Vision 2010, 2010 <https://engineering.purdue.edu/RVL/Publications/Aeschliman2010ANovel.pdf>

---

|            |                                                                              |
|------------|------------------------------------------------------------------------------|
| vmm_fit_em | <i>Estimate von Mises Mixture parameters using Expectation Maximization.</i> |
|------------|------------------------------------------------------------------------------|

---

**Description**

Estimates parameters for univariate von Mises mixture using Expectation Maximization algorithm.

**Usage**

```
vmm_fit_em( x, p, epsilon = c( 0.000001, 0.000001, 0.000001 ),
            debug = FALSE, implementation = "C" )
```





**Arguments**

|                |                                                                                                                                                                                                                                                                                                                                     |
|----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x              | data vector                                                                                                                                                                                                                                                                                                                         |
| p              | initialization vector of $3*n$ parameters, where $n$ is number of mixture components. Structure of p vector is $p = c( A1, A2, ..., An, mu1, mu2, ..., mun, k1, k2, ..., kn )$ , where $A_i$ is the proportion of $i$ -th component, $mu_i$ is the center of $i$ -th component and $k_i$ is the concentration of $i$ -th component. |
| epsilon        | tolerance threshold for convergence                                                                                                                                                                                                                                                                                                 |
| debug          | flag to turn the debug prints on/off.                                                                                                                                                                                                                                                                                               |
| implementation | flag to switch between C (default) and R implementations.                                                                                                                                                                                                                                                                           |

**Value**

Vector of mixture parameters, whose structure is the same as of input parameter's p.

**Author(s)**

Andrius Merkys

**References**

Banerjee et al. Expectation Maximization for Clustering on Hyperspheres (2003), manuscript, accessible on: <https://web.archive.org/web/20130120061240/http://www.lans.ece.utexas.edu/~abanerjee/papers/05/banerjee05a.pdf>

---

|                 |                                                                              |
|-----------------|------------------------------------------------------------------------------|
| vmm_init_vector | <i>Estimate von Mises Mixture parameters using Expectation Maximization.</i> |
|-----------------|------------------------------------------------------------------------------|

---

**Description**

Estimate an initialization vector for von Mises mixture fitting via Expectation Maximization. Proportions are set to equal, centers are equispaced through the whole domain of input sample, and concentrations are set to  $(m/(12*180))^2$ .

**Usage**

```
vmm_init_vector( m, implementation = "C" )
```

**Arguments**

|                |                                                           |
|----------------|-----------------------------------------------------------|
| m              | number of mixture components                              |
| implementation | flag to switch between C (default) and R implementations. |

**Value**

Parameter vector of  $3*n$  parameters, where  $n$  is number of mixture components. Structure of  $p$  vector is  $p = c( A1, A2, ..., An, mu1, mu2, ..., mun, k1, k2, ..., kn )$ , where  $A_i$  is the proportion of  $i$ -th component,  $\mu_i$  is the center of  $i$ -th component and  $k_i$  is the concentration of  $i$ -th component.

**Author(s)**

Andrius Merkys

---

wmedian

*Calculate Weighted Median.*

---

**Description**

Calculated weighted median.

**Usage**

```
wmedian( x, w, start = 1, end = length( x ) )
```

**Arguments**

|                    |                                         |
|--------------------|-----------------------------------------|
| <code>x</code>     | sample vector                           |
| <code>w</code>     | weights vector                          |
| <code>start</code> | start index (default: 1)                |
| <code>end</code>   | end index (default: last index in $x$ ) |

**Value**

Median

**Author(s)**

Andrius Merkys

**References**

Users of Wikipedia. Weighted median. [https://en.wikipedia.org/w/index.php?title=Weighted\\_median&oldid=690896947](https://en.wikipedia.org/w/index.php?title=Weighted_median&oldid=690896947)

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