Package ‘MittagLeffleR’

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Type Package

Title Mittag-Leffler Family of Distributions

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Description Implements the Mittag-Leffler function, distribution, random variate generation, and estimation. Based on the Laplace-Inversion algorithm by Garrappa, R. (2015) <doi:10.1137/140971191>.

URL https://strakaps.github.io/MittagLeffleR/

BugReports https://github.com/strakaps/MittagLeffleR/issues

License GPL (>= 2)

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MittagLeffleR-package *Mittag-Leffler family of distributions*

Description

A generalization of the exponential distribution. Contains

- the Mittag-Leffler function `mlf`
- distributions (`dml`, `pml`, `qml`) and random variate generation (`rml`)
- a log-moment estimator (`logMomentEstimator`), and maximum likelihood estimator (`mlmle`)

Details

- Plots of the Mittag-Leffler distributions
- Details of Mittag-Leffler random variate generation
- Probabilities and Quantiles

Also see the package web page at `https://strakaps.github.io/MittagLeffleR/reference/index.html`

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See Also

Useful links:

- `https://strakaps.github.io/MittagLeffleR/`
- Report bugs at `https://github.com/strakaps/MittagLeffleR/issues`

`logMomentEstimator` *Log-Moments Estimator for the Mittag-Leffler Distribution (Type 1).*

Description

Tail and scale parameter of the Mittag-Leffler distribution are estimated by matching with the first two empirical log-moments (see Cahoy et al., doi: 10.1016/j.jspi.2010.04.016).

Usage

```
logMomentEstimator(x, alpha = 0.05)
```

Arguments

`x`	A vector of non-negative data.
`alpha`	Confidence intervals are calculated at level 1 - alpha.

Value

A named vector with entries (nu, delta, nuLo, nuHi, deltaLo, deltaHi) where nu is the tail parameter and delta the scale parameter of the Mittag-Leffler distribution, with confidence intervals (nuLo, nuHi) resp. (deltaLo, deltaHi).

References

Cahoy, D. O., Uchaikin, V. V., & Woyczynski Wojbor, W. A. (2010). Parameter estimation for fractional Poisson processes. Journal of Statistical Planning and Inference, 140(11), 3106–3120. doi: 10.1016/j.jspi.2010.04.016

Cahoy, D. O. (2013). Estimation of Mittag-Leffler Parameters. Communications in Statistics - Simulation and Computation, 42(2), 303–315. doi: 10.1080/03610918.2011.640094

Examples

```
logMomentEstimator(rml(n = 1000, scale = 0.03, tail = 0.84), alpha=0.95)
```

`Mittag-Leffler` *Distribution functions and random number generation.*

Description

Probability density, cumulative distribution function, quantile function and random variate generation for the two types of Mittag-Leffler distribution. The Laplace inversion algorithm by Garrappa is used for the pdf and cdf (see `https://www.mathworks.com/matlabcentral/fileexchange/48154-the-mittag-leffler-function`).

Usage

```
dml(x, tail, scale = 1, log = FALSE, second.type = FALSE)

pml(q, tail, scale = 1, second.type = FALSE, lower.tail = TRUE, log.p = FALSE)

qml(p, tail, scale = 1, second.type = FALSE, lower.tail = TRUE, log.p = FALSE)

rml(n, tail, scale = 1, second.type = FALSE)
```

Arguments

`x, q`	vector of quantiles.
`tail`	tail parameter.
`scale`	scale parameter.
`log, log.p`	logical; if TRUE, probabilities p are given as log(p).
`second.type`	logical; if FALSE (default), first type of Mittag-Leffler distribution is assumed.
`lower.tail`	logical; if TRUE, probabilities are $P[X \le x]$ otherwise, $P[X > x]$
`p`	vector of probabilities.
`n`	number of random draws.

Details

The Mittag-Leffler function `mlf` defines two types of probability distributions:

The **first type** of Mittag-Leffler distribution assumes the Mittag-Leffler function as its tail function, so that the CDF is given by

$$F(q; \alpha, \tau) = 1 - E_{\alpha,1}(-(q/\tau)^\alpha)$$

for $q \ge 0$, tail parameter $0 < \alpha \le 1$, and scale parameter $\tau > 0$. Its PDF is given by

$$f(x; \alpha, \tau) = x^{\alpha-1} E_{\alpha,\alpha}[-(x/\tau)^\alpha]/\tau^\alpha.$$

As α approaches 1 from below, the Mittag-Leffler converges (weakly) to the exponential distribution. For $0 < \alpha < 1$, it is (very) heavy-tailed, i.e. has infinite mean.

The **second type** of Mittag-Leffler distribution is defined via the Laplace transform of its density f:

$$\int_0^\infty \exp(-sx) f(x; \alpha, 1) dx = E_{\alpha,1}(-s)$$

It is light-tailed, i.e. all its moments are finite. At scale τ, its density is

$$f(x; \alpha, \tau) = f(x/\tau; \alpha, 1)/\tau.$$

Value

`dml` returns the density, `pml` returns the distribution function, `qml` returns the quantile function, and `rml` generates random variables.

References

Haubold, H. J., Mathai, A. M., & Saxena, R. K. (2011). Mittag-Leffler Functions and Their Applications. Journal of Applied Mathematics, 2011, 1–51. doi: 10.1155/2011/298628

Mittag-Leffler distribution. (2017, May 3). In Wikipedia, The Free Encyclopedia. https://en.wikipedia.org/w/index.php?title=Mittag-Leffler_distribution&oldid=778429885

Examples

```
dml(1, 0.8)
dml(1, 0.6, second.type=TRUE)
pml(2, 0.7, 1.5)
qml(p = c(0.25, 0.5, 0.75), tail = 0.6, scale = 100)
rml(10, 0.7, 1)
```

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mlf *Mittag-Leffler Function.*

Description

The generalized (two-parameter) Mittag-Leffer function is defined by the power series

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} z^k / \Gamma(\alpha k + \beta)$$

for complex z and complex α, β with $Real(\alpha) > 0$ (only implemented for real valued parameters).

Usage

```
mlf(z, a, b = 1, g = 1)
```

Arguments

`z`	The argument (real-valued)
`a, b, g`	Parameters of the Mittag-Leffler distribution; see Garrappa

Value

`mlf` returns the value of the Mittag-Leffler function.

References

Garrappa, R. (2015). Numerical Evaluation of Two and Three Parameter Mittag-Leffler Functions. SIAM Journal on Numerical Analysis, 53(3), 1350–1369. doi: 10.1137/140971191

The Mittag-Leffler function. MathWorks File Exchange. https://au.mathworks.com/matlabcentral/fileexchange/48154-the-mittag-leffler-function

Examples

```
mlf(2,0.7)
```

---

mlmle *Maximum Likelihood Estimation of the Mittag-Leffler distribution*

Description

Optimizes the bivariate loglikelihood of the Mittag-Leffler distribution via `optim`. Uses `logMomentEstimator` for initial parameter values.

Usage

```
mlmle(data, ...)
```

Arguments

`data`	Vector of class "numeric"
`...`	Additional parameters passed on to `optim`.

Value

The output of `optim`.

Examples

```
library(magrittr)
rml(n = 100, tail = 0.8, scale = 1000) %>% mlmle()
```

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