

# Package ‘cocons’

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**Type** Package

**Title** Covariate-Based Covariance Functions for Nonstationary Spatial Modeling

**Version** 0.1.4

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## Description

Estimation, prediction, and simulation of nonstationary Gaussian process with modular covariate-based covariance functions.

Sources of nonstationarity, such as spatial mean, variance, geometric anisotropy, smoothness, and nugget, can be considered based on spatial characteristics.

An induced compact-supported nonstationary covariance function is provided, enabling fast and memory-efficient computations when handling densely sampled domains.

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**LazyData** true

**License** GPL (>= 3)

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methods, knitr

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## Contents

|  |    |
|--|----|
| cocons-package . . . . .                   | 3  |
| coco . . . . .                             | 3  |
| coco-class . . . . .                       | 6  |
| cocoOptim . . . . .                        | 6  |
| cocoPredict . . . . .                      | 8  |
| cocoSim . . . . .                          | 10 |
| cov_rns . . . . .                          | 12 |
| cov_rns_classic . . . . .                  | 12 |
| cov_rns_pred . . . . .                     | 13 |
| cov_rns_taper . . . . .                    | 14 |
| cov_rns_taper_pred . . . . .               | 14 |
| getAIC . . . . .                           | 15 |
| getBIC . . . . .                           | 16 |
| getBoundaries . . . . .                    | 16 |
| getBoundariesV2 . . . . .                  | 17 |
| getBoundariesV3 . . . . .                  | 18 |
| getCIs . . . . .                           | 19 |
| getCovMatrix . . . . .                     | 19 |
| getCRPS . . . . .                          | 20 |
| getDensityFromDelta . . . . .              | 21 |
| getDesignMatrix . . . . .                  | 22 |
| getEstims . . . . .                        | 22 |
| getHessian . . . . .                       | 23 |
| getLoglik . . . . .                        | 23 |
| getLogScore . . . . .                      | 24 |
| getModelLists . . . . .                    | 25 |
| getModHess . . . . .                       | 25 |
| GetNeg2loglikelihood . . . . .             | 26 |
| GetNeg2loglikelihoodProfile . . . . .      | 27 |
| GetNeg2loglikelihoodREML . . . . .         | 28 |
| GetNeg2loglikelihoodTaper . . . . .        | 29 |
| GetNeg2loglikelihoodTaperProfile . . . . . | 30 |
| getScale . . . . .                         | 31 |
| getSpatEffects . . . . .                   | 31 |
| getSpatMean . . . . .                      | 32 |
| holes . . . . .                            | 32 |
| holes_bm . . . . .                         | 33 |
| is.formula . . . . .                       | 34 |
| plot,coco,missing-method . . . . .         | 34 |
| plotOptimInfo . . . . .                    | 35 |
| stripes . . . . .                          | 36 |
| summary . . . . .                          | 36 |

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 cocons-package

*Covariate-based Covariance Functions for Nonstationary Gaussian Processes*


---

## Description

Provides routines and methods for estimating and predicting nonstationary Gaussian process models with modular covariate-based covariance functions. Several sources of nonstationarity can be modeled based on spatial information, including a spatial mean, marginal standard deviation, local geometric anisotropy, local nugget, and spatially varying smoothness. Each of these components is modeled separately. An induced compact-supported nonstationary covariance function is provided to speed up computations when handling densely sampled domains. Model parameters are estimated via maximum likelihood (and flavours of it, such as penalized and profile maximum likelihood). A variety of functions are also included to compute prediction metrics and to visualize, simulate, and summarize these types of models. Details of the models can be found in the vignette and in [coco](#).

## Disclaimer

This package is provided "as is" without warranty of any kind, either express or implied. Backwards compatibility will not be offered until later versions.

## Author(s)

Federico Blasi [aut, cre], <federico.blasi@gmail.com>

## Examples

```
## Not run:
  vignette("cocons", package = "cocons")
  methods(class = "coco")

## End(Not run)
```

---

 coco

*Creates a coco S4 object*


---

## Description

Creates an S4 object of class `coco`, which is the centerpiece of the **cocons** package. The function provides a set of consistency checks for ensuring the suitability of the different objects involved.

## Usage

```
coco(type, data, locs, z, model.list, info, output = list())
```

## Arguments

|                         |  |
|-------------------------|--|
| <code>type</code>       | (character) One of two available types "dense" or "sparse". See description.   |
| <code>data</code>       | (data.frame) A data.frame with covariates information, where <code>colnames(data)</code> matches <code>model.list</code> specification.  |
| <code>locs</code>       | (matrix) A matrix with spatial locations.  |
| <code>z</code>          | (vector or matrix) A matrix of $n \times r$ response realizations, one realization per column. When considering only one realization, a vector can also be provided.                                     |
| <code>model.list</code> | (list) A list specifying a model for each source of nonstationarity.   |
| <code>info</code>       | (list or NULL) A list specifying characteristics of the coco object.   |
| <code>output</code>     | (list or NULL) Empty or the resulting object from running <code>optimParallel</code> , adding to this a list with boundary information (see <code>getBoundaries</code> to check the expected structure). |

## Details

Two types of coco objects are available, each assuming a different type of covariance matrix for the Gaussian process. Type "dense" builds dense covariance matrices (non zero elements), while type "sparse" builds sparse covariance matrices by tapering the dense covariance matrix with a compact isotropic compact-supported correlation matrix [1]. Type "sparse" allows a set of efficient algorithms, thus making it more suitable for large sample sizes.

An important component of the coco S4 class is the `model.list` specification, involving individual formulas provided as a list, where each of them specifies a covariate-based parametric model for a specific source of nonstationarity. It involves "mean" for the spatial mean, the "std.dev" for the marginal standard deviation, "scale", "aniso" and "tilt", each of them shaping specific aspects of the local spatial geometrically anisotropy structure, "smooth" handling local smoothness, and "nugget" handling the local nugget effect. The models are defined as:

| Source         | Related to          | Description                   | Model   |
|----------------|---------------------|-------------------------------|---|
| <i>mean</i>    | $\mu$               | Spatial mean function         | $\mathbf{X}_1\beta$                                     |
| <i>std.dev</i> | $\sigma^X$          | Marginal standard deviation   | $\exp(0.5\mathbf{X}_2\alpha)$                           |
| <i>scale</i>   | $\Sigma^X$          | Local scale                   | $\exp(\mathbf{X}_3\theta_1)$                            |
| <i>aniso</i>   | $\Sigma^X$          | Local geometric anisotropy    | $\exp(\mathbf{X}_4\theta_2)$                            |
| <i>tilt</i>    | $\Sigma^X$          | (Restricted) local tilt       | $\cos(\text{logit}^{-1}(\mathbf{X}_5\theta_3))$         |
| <i>smooth</i>  | $\nu^X$             | Local smoothness              | $(\nu_u - \nu_l)/(1 + \exp(-\mathbf{X}_6\phi)) + \nu_l$ |
| <i>nugget</i>  | $\sigma_\epsilon^X$ | Local micro-scale variability | $\exp(\mathbf{X}_7\zeta)$                               |

where  $\beta$ ,  $\alpha$ ,  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$ ,  $\phi$ , and  $\zeta$  are the parameter vectors of each model,  $\nu_l$ , and  $\nu_u$  are the lower and upper bounds limiting the range of variation of the spatially-varying smoothness, and where  $\mathbf{X}_\ell$  relates to the design matrix defined by the specific models for each of the source of nonstationarity.

Lastly, arguments for the "info" list argument involve:

- "lambda": (numeric) a positive scalar specifying the regularization parameter. Larger values penalizes highly-smoothed long-tailed covariance functions.
- "smooth.limits": (numeric vector) specifying the range of variation for the spatially varying smoothness (e.g. `c(0.5, 2.5)`).

- "taper": (numeric) specifying the desired taper function from the spam package (only for "sparse" coco objects).
- "delta": (numeric) specifying the taper range/scale (only for "sparse" coco objects).
- "skip.scale": (integer vector) By default, all covariates are scaled. skip.scale allows to specify the index of those variables in data that should not be scaled during the optimization.

### Value

(S4) An S4 object of class coco.

### Author(s)

Federico Blasi

### References

[1] Furrer, Reinhard, Marc G. Genton, and Douglas Nychka. "Covariance tapering for interpolation of large spatial datasets." *Journal of Computational and Graphical Statistics* 15.3 (2006): 502-523.

### See Also

[spam::cov.wend1\(\)](#)

### Examples

```
## Not run:
locs <- expand.grid(seq(0,1,length.out = 10),
  seq(0,1,length.out = 10))

toydata <- data.frame('x' = locs[,1])

set.seed(1)
z <- rnorm(100)

model.list <- list('mean' = 0,
  'std.dev' = formula( ~ 1),
  'scale' = formula( ~ 1 + x),
  'aniso' = 0,
  'tilt' = 0,
  'smooth' = 3/2,
  'nugget' = -Inf)

coco_object <- coco(type = 'dense',
  data = toydata,
  locs = as.matrix(locs),
  z = z,
  model.list = model.list)

coco_object
```

```
## End(Not run)
```

---

|            |   |
|------------|---|
| coco-class | <i>An S4 class to store information</i> |
|------------|---|

---

### Description

An S4 class to store information

### Slots

type (character) One of two available types "dense" or "sparse". See description.

data (data.frame) A data.frame with covariates information, where colnames(data) matches model.list specification

locs (numeric matrix) a matrix with locs matching data

z (numeric matrix) A matrix of dimension n x p with response values

model.list (list) A list specifying a model for each aspect of the spatial structure.

info (list) a list with information about the coco object

output (list) if building an already fitted coco object (not the standard approach), then requires an output from Optimparallel output, including as well boundaries, etc.

### Author(s)

Federico Blasi

---

|           |                                   |
|-----------|-----------------------------------|
| cocoOptim | <i>Optimizer for coco objects</i> |
|-----------|-----------------------------------|

---

### Description

Estimation the spatial model parameters using the L-BFGS-B optimizer [1].

### Usage

```
cocoOptim(coco.object, boundaries = list(), ncores = "auto", safe = TRUE,
optim.type, optim.control)
```

**Arguments**

|               |  |
|---------------|--|
| coco.object   | (S4) A <a href="#">coco</a> object.  |
| boundaries    | (list) If provided, a list containing lower, initial, and upper values for the parameters, as defined by <a href="#">getBoundaries</a> . If not provided, these values are automatically computed with global lower and upper bounds set to -2 and 2.  |
| ncores        | (character or integer) The number of threads to use for the optimization. If set to "auto", the number of threads is chosen based on system capabilities or a fraction of the available cores.   |
| safe          | (logical) If TRUE, the function avoids Cholesky decomposition errors due to ill-posed covariance matrices by returning a pre-defined large value. Defaults to TRUE. <ul style="list-style-type: none"> <li>"ml": Classical Maximum Likelihood estimation.</li> <li>"pml": Profile Maximum Likelihood, factoring out the spatial trend for dense objects or the global marginal variance parameter for sparse objects.</li> </ul> |
| optim.type    | (character) The optimization approach. Options include:  |
| optim.control | (list) A list of settings to be passed to the <a href="#">optimParallel</a> function [2].  |

**Value**

(S4) An optimized S4 object of class coco.

**Author(s)**

Federico Blasi

**References**

- [1] Byrd, Richard H., et al. "A limited memory algorithm for bound constrained optimization." SIAM Journal on scientific computing 16.5 (1995): 1190-1208.
- [2] Gerber, Florian, and Reinhard Furrer. "optimParallel: An R package providing a parallel version of the L-BFGS-B optimization method." R Journal 11.1 (2019): 352-358.

**See Also**

[\[optimParallel\]](#)

**Examples**

```
## Not run:
model.list <- list('mean' = 0,
                  'std.dev' = formula(~ 1 + cov_x + cov_y),
                  'scale' = formula(~ 1 + cov_x + cov_y),
                  'aniso' = 0,
                  'tilt' = 0,
                  'smooth' = 3/2,
                  'nugget' = -Inf)
```

```

coco_object <- coco(type = 'dense',
                   data = holes[[1]][1:100,],
                   locs = as.matrix(holes[[1]][1:100,1:2]),
                   z = holes[[1]][1:100,]$z,
                   model.list = model.list)

optim_coco <- cocoOptim(coco_object,
                      boundaries = getBoundaries(coco_object,
                                                  lower.value = -3, 3))

plotOptimInfo(optim_coco)

plot(optim_coco)

plot(optim_coco, type = 'ellipse')

plot(optim_coco, type = 'correlations', index = c(2,3,5))

summary(optim_coco)

getEstims(optim_coco)

## End(Not run)

```

---

cocoPredict

*Prediction for coco objects*


---

### Description

Computes the conditional expectation and standard errors based on the conditional Gaussian distribution for nonstationary spatial models.

### Usage

```
cocoPredict(coco.object, newdataset, newlocs, type = 'mean', ...)
```

### Arguments

|             |   |
|-------------|---|
| coco.object | (S4) A fitted <code>coco</code> object.   |
| newdataset  | (data.frame) A data.frame containing the covariates present in <code>model.list</code> at the prediction locations.   |
| newlocs     | (matrix) A matrix specifying the prediction locations, matching <code>newdataset</code> index.  |
| type        | (character) Specifies whether to return only the point prediction ('mean') or both the point prediction and prediction standard errors ('pred').  |
| ...         | Additional arguments. If <code>coco.object</code> contains multiple realizations, the argument <code>index.pred</code> can be used to specify which realization of <code>coco.object@z</code> should be used for predictions. |

**Value**

A list containing:

- `systematic`: The systematic component of the conditional expectation.
- `stochastic`: The stochastic component of the conditional expectation.
- `sd.pred`: The standard errors, when `type = 'pred'` is specified.

**Author(s)**

Federico Blasi

**Examples**

```
## Not run:

# Stationary model

model.list_stat <- list('mean' = 0,
  'std.dev' = formula(~ 1),
  'scale' = formula(~ 1),
  'aniso' = 0,
  'tilt' = 0,
  'smooth' = 3/2,
  'nugget' = -Inf)

model.list_ns <- list('mean' = 0,
  'std.dev' = formula(~ 1 + cov_x + cov_y),
  'scale' = formula(~ 1 + cov_x + cov_y),
  'aniso' = 0,
  'tilt' = 0,
  'smooth' = 3/2,
  'nugget' = -Inf)

coco_object <- coco(type = 'dense',
  data = holes[[1]][1:100, ],
  locs = as.matrix(holes[[1]][1:100, 1:2]),
  z = holes[[1]][1:100, ]$z,
  model.list = model.list_stat)

optim_coco_stat <- cocoOptim(coco_object,
  boundaries = getBoundaries(coco_object,
  lower.value = -3, 3))

coco_preds_stat <- cocoPredict(optim_coco_stat, newdataset = holes[[2]],
  newlocs = as.matrix(holes[[2]][, 1:2]),
  type = "pred")

# Update model
coco_object@model.list <- model.list_ns
```

```

optim_coco_ns <- cocoOptim(coco_object,
  boundaries = getBoundaries(coco_object,
    lower.value = -3, 3))

coco_preds_ns <- cocoPredict(optim_coco_ns, newdataset = holes[[2]],
  newlocs = as.matrix(holes[[2]][, 1:2]),
  type = "pred")

par(mfrow = c(1, 3))

fields::quilt.plot(main = "full data", holes[[1]][, 1:2],
  holes[[1]]$z, xlim = c(-1, 1), ylim = c(-1, 1))

fields::quilt.plot(main = "stationary se", holes[[2]][, 1:2],
  coco_preds_stat$sd.pred, xlim = c(-1, 1), ylim = c(-1, 1))
fields::quilt.plot(main = "nonstationary se", holes[[2]][, 1:2],
  coco_preds_ns$sd.pred, xlim = c(-1, 1), ylim = c(-1, 1))

## End(Not run)

```

---

cocoSim

---

*Marginal and conditional simulation of nonstationary Gaussian processes*


---

## Description

draw realizations of stationary and nonstationary Gaussian processes with covariate-based covariance functions.

## Usage

```

cocoSim(coco.object, pars, n, seed, standardize,
  type = 'classic', sim.type = NULL, cond.info = NULL)

```

## Arguments

|             |  |
|-------------|--|
| coco.object | (S4) A <code>coco</code> object.   |
| pars        | (numeric vector or NULL) A vector of parameter values associated with <code>model.list</code> . If <code>coco.object</code> is a fitted object, and <code>pars</code> is NULL, it get <code>pars</code> from <code>coco.object@output\$pars</code> (and also sets 'type' to 'diff'). |
| n           | (integer) Number of realizations to simulate.  |
| seed        | (integer or NULL) Seed for random number generation. Defaults to NULL.   |
| standardize | (logical) Indicates whether the provided covariates should be standardized (TRUE) or not (FALSE). Defaults to TRUE.  |

|           |   |
|-----------|---|
| type      | (character) Specifies whether the parameters follow a classical parameterization ('classic') or a difference parameterization ('diff'). Defaults to 'classic'. For sparse coco objects, only 'diff' is allowed. |
| sim.type  | (character) If set to 'cond', a conditional simulation is performed.  |
| cond.info | (list) A list containing additional information required for conditional simulation.  |

### Details

The argument `sim.type = 'cond'` specifies a conditional simulation, requiring `cond.info` to be provided. `cond.info` is a list including `newdataset`, a `data.frame` containing covariates present in `model.list` at the simulation locations, and `newlocs`, a matrix specifying the locations corresponding to the simulation, with indexing that matches `newdataset`.

The argument `type = 'classic'` assumes a simplified parameterization for the covariance function, with log-parameterizations applied to the parameters `std.dev`, `scale`, and `smooth`.

### Value

(matrix) a matrix `dim(data)[1] x n`.

### Author(s)

Federico Blasi

### See Also

[coco](#)

### Examples

```
## Not run:

model.list <- list('mean' = 0,
                  'std.dev' = formula(~ 1 + cov_x + cov_y),
                  'scale' = formula(~ 1 + cov_x + cov_y),
                  'aniso' = 0,
                  'tilt' = 0,
                  'smooth' = 0.5,
                  'nugget' = -Inf)

coco_object <- coco(type = 'dense',
                  data = holes[[1]][1:1000,],
                  locs = as.matrix(holes[[1]][1:1000,1:2]),
                  z = holes[[1]][1:1000,]$,
                  model.list = model.list)

coco_sim <- cocoSim(coco.object = coco_object,
                  pars = c(0,0.25,0.25, # pars related to std.dev
                          log(0.25),1,-1), # pars related to scale
                  n = 1,
```

```

        standardize = TRUE)

fields::quilt.plot(coco_object@locs,coco_sim)

## End(Not run)

```

---

cov\_rns                      *Dense covariance function (difference parameterization)*

---

### Description

Dense covariance function (difference parameterization)

### Usage

```
cov_rns(theta, locs, x_covariates, smooth_limits)
```

### Arguments

|               |                         |
|---------------|-------------------------|
| theta         | vector of parameters    |
| locs          | a matrix with locations |
| x_covariates  | design data.frame       |
| smooth_limits | smooth limits           |

### Value

dense covariance matrix

---

cov\_rns\_classic                      *Dense covariance function (classic parameterization)*

---

### Description

Dense covariance function (classic parameterization)

### Usage

```
cov_rns_classic(theta, locs, x_covariates)
```

### Arguments

|              |                         |
|--------------|-------------------------|
| theta        | vector of parameters    |
| locs         | a matrix with locations |
| x_covariates | design data.frame       |

**Value**

dense covariance matrix with classic parameterization

---

|              |                                  |
|--------------|----------------------------------|
| cov_rns_pred | <i>Dense covariance function</i> |
|--------------|----------------------------------|

---

**Description**

Dense covariance function

**Usage**

```
cov_rns_pred(  
  theta,  
  locs,  
  locs_pred,  
  x_covariates,  
  x_covariates_pred,  
  smooth_limits  
)
```

**Arguments**

|                   |   |
|-------------------|---|
| theta             | vector of parameters                      |
| locs              | a matrix with locations                   |
| locs_pred         | a matrix with prediction locations        |
| x_covariates      | design data.frame                         |
| x_covariates_pred | design data.frame at prediction locations |
| smooth_limits     | smooth limits                             |

**Value**

dense covariance matrix

---

|               |                                   |
|---------------|-----------------------------------|
| cov_rns_taper | <i>Sparse covariance function</i> |
|---------------|-----------------------------------|

---

**Description**

Sparse covariance function

**Usage**

```
cov_rns_taper(
  theta,
  locs,
  x_covariates,
  colindices,
  rowpointers,
  smooth_limits
)
```

**Arguments**

|               |                         |
|---------------|-------------------------|
| theta         | vector of parameters    |
| locs          | a matrix with locations |
| x_covariates  | design data.frame       |
| colindices    | from spam object        |
| rowpointers   | from spam object        |
| smooth_limits | smooth limits           |

**Value**

sparse covariance matrix between locs and pred\_locs

---

|                    |                                   |
|--------------------|-----------------------------------|
| cov_rns_taper_pred | <i>Sparse covariance function</i> |
|--------------------|-----------------------------------|

---

**Description**

Sparse covariance function

**Usage**

```
cov_rns_taper_pred(
  theta,
  locs,
  locs_pred,
  x_covariates,
  x_covariates_pred,
  colindices,
  rowpointers,
  smooth_limits
)
```

**Arguments**

|                   |   |
|-------------------|---|
| theta             | vector of parameters                      |
| locs              | a matrix with locations                   |
| locs_pred         | a matrix with prediction locations        |
| x_covariates      | design data.frame                         |
| x_covariates_pred | design data.frame at prediction locations |
| colindices        | from spam object                          |
| rowpointers       | from spam object                          |
| smooth_limits     | smooth limits                             |

**Value**

sparse covariance matrix at locs

---

|        |                     |
|--------|---------------------|
| getAIC | <i>Retrieve AIC</i> |
|--------|---------------------|

---

**Description**

Retrieve the Akaike information criterion from a fitted coco object.

**Usage**

```
getAIC(coco.object)
```

**Arguments**

|             |                               |
|-------------|-------------------------------|
| coco.object | (S4) a fitted coco S4 object. |
|-------------|-------------------------------|

**Value**

(numeric) the associated AIC value

**Author(s)**

Federico Blasi

---

`getBIC`*Retrieve BIC*

---

**Description**

Retrieve BIC from a fitted coco object.

**Usage**`getBIC(coco.object)`**Arguments**`coco.object` (S4) a fitted coco S4 object.**Value**

(numeric) the associated BIC value

**Author(s)**

Federico Blasi

---

`getBoundaries`*Simple build of boundaries*

---

**Description**

provides a generic set of upper and lower bounds for the L-BFGS-B routine

**Usage**`getBoundaries(x, lower.value, upper.value)`**Arguments**`x` (S4) or (list) a coco.object or a par.pos list (as output from [getDesignMatrix](#))`lower.value` (numeric vector) if provided, provides a vector filled with values lower.value.`upper.value` (numeric vector) if provided, provides a vector filled with values upper.value.**Value**

(list) a list with boundaries and simple init values for the optim L-BFGS-B routine

**Author(s)**

Federico Blasi

---

|                 |  |
|-----------------|--|
| getBoundariesV2 | <i>Simple build of boundaries (v2)</i> |
|-----------------|--|

---

**Description**

provides a generic set of upper and lower bounds for the L-BFGS-B routine

**Usage**

```
getBoundariesV2(coco.object, mean.limits, std.dev.limits,  
scale.limits, aniso.limits, tilt.limits, smooth.limits, nugget.limits)
```

**Arguments**

|                |   |
|----------------|---|
| coco.object    | (S4) a coco object.   |
| mean.limits    | (numeric vector) a vector of c(lower,init,upper) values for the associated param. |
| std.dev.limits | (numeric vector) a vector of c(lower,init,upper) values for the associated param. |
| scale.limits   | (numeric vector) a vector of c(lower,init,upper) values for the associated param. |
| aniso.limits   | (numeric vector) a vector of c(lower,init,upper) values for the associated param. |
| tilt.limits    | (numeric vector) a vector of c(lower,init,upper) values for the associated param. |
| smooth.limits  | (numeric vector) a vector of c(lower,init,upper) values for the associated param. |
| nugget.limits  | (numeric vector) a vector of c(lower,init,upper) values for the associated param. |

**Value**

(list) a list with boundaries for the optim L-BFGS-B routine

**Author(s)**

Federico Blasi

---

`getBoundariesV3`*Simple build of boundaries (v3)*

---

**Description**

provides a generic set of upper and lower bounds for the L-BFGS-B routine

**Usage**

```
getBoundariesV3(coco.object, mean.limits, global.lower,  
std.dev.max.effects,  
scale.max.effects, aniso.max.effects, tilt.max.effects,  
smooth.max.effects, nugget.max.effects)
```

**Arguments**

`coco.object` (S4) a coco object.

`mean.limits` (numeric vector) a vector of c(lower,init,upper) values for the associated param.

`global.lower` (numeric vector) a vector of c(lower,init,upper) values for the associated param.

`std.dev.max.effects`  
(numeric vector) a vector of c(lower,init,upper) values for the associated param.

`scale.max.effects`  
(numeric vector) a vector of c(lower,init,upper) values for the associated param.

`aniso.max.effects`  
(numeric vector) a vector of c(lower,init,upper) values for the associated param.

`tilt.max.effects`  
(numeric vector) a vector of c(lower,init,upper) values for the associated param.

`smooth.max.effects`  
(numeric vector) a vector of c(lower,init,upper) values for the associated param.

`nugget.max.effects`  
(numeric vector) a vector of c(lower,init,upper) values for the associated param.

**Value**

(list) a list with boundaries for the optim L-BFGS-B routine

**Author(s)**

Federico Blasi

---

|        |   |
|--------|---|
| getCIs | <i>Compute approximate confidence intervals for a coco object</i> |
|--------|---|

---

**Description**

Compute approximate confidence intervals for a (fitted) coco object.

**Usage**

```
getCIs(coco.object, inv.hess, alpha = 0.95)
```

**Arguments**

|             |   |
|-------------|---|
| coco.object | (S4) a fitted coco S4 object.                                 |
| inv.hess    | (matrix) Inverse of the Hessian. <a href="#">getHessian</a> . |
| alpha       | (numeric) confidence level.                                   |

**Value**

(numeric matrix) a matrix with approximate confidence intervals for each parameter in the model.

**Author(s)**

Federico Blasi

---

|              |   |
|--------------|---|
| getCovMatrix | <i>Covariance matrix for "coco" class</i> |
|--------------|---|

---

**Description**

Compute the covariance matrix of coco.object.

**Usage**

```
getCovMatrix(coco.object, type = 'global', index = NULL)
```

**Arguments**

|             |  |
|-------------|--|
| coco.object | (S4) a fitted <a href="#">coco()</a> object.   |
| type        | (character) whether 'global' to retrieve the regular covariance matrix, or 'local' to retrieve global covariance. based on the local aspects of a specific location (not implemented yet). |
| index       | (integer) index to perform local covariance matrix (not implemented yet).  |

**Value**

(matrix or S4) a  $n \times n$  covariance matrix (for 'dense' coco objects) or a S4 spam object (for 'sparse' coco objects).

**Author(s)**

Federico Blasi

**Examples**

```
## Not run:
model.list <- list('mean' = 0,
                  'std.dev' = formula(~ 1 + cov_x + cov_y),
                  'scale' = formula(~ 1 + cov_x + cov_y),
                  'aniso' = 0,
                  'tilt' = 0,
                  'smooth' = 3/2,
                  'nugget' = -Inf)

coco_object <- coco(type = 'dense',
                  data = holes[[1]][1:100,],
                  locs = as.matrix(holes[[1]][1:100,1:2]),
                  z = holes[[1]][1:100,]z,
                  model.list = model.list)

optim_coco <- cocoOptim(coco_object,
                      boundaries = getBoundaries(coco_object,
                                                  lower.value = -3, 3))

getCovMatrix(optim_coco)

## End(Not run)
```

---

getCRPS

*Based on a set of predictions computes the Continuous Ranked Probability Score*

---

**Description**

Retrieves the Continuous Ranked Probability Score (CRPS) [1].

**Usage**

```
getCRPS(z.pred, mean.pred, sd.pred)
```

**Arguments**

z.pred (numeric vector).  
mean.pred (numeric vector).  
sd.pred (numeric vector).

**Value**

(numeric vector) retrieves CRPS.

**Author(s)**

Federico Blasi

**References**

[1] Gneiting, Tilmann, and Adrian E. Raftery. "Strictly proper scoring rules, prediction, and estimation." Journal of the American statistical Association 102.477 (2007): 359-378.

---

getDensityFromDelta *Based on a specific taper scale (delta), retrieves the density of the covariance matrix.*

---

**Description**

Based on a specific taper scale (delta), retrieves the density of the covariance matrix.

**Usage**

```
getDensityFromDelta(coco.object, delta)
```

**Arguments**

coco.object (S4) a fitted `coco()` object.  
delta (numeric) a delta taper scale (delta).

**Value**

(numeric vector) the associate density of the tapered covariance matrix.

**Author(s)**

Federico Blasi

---

getDesignMatrix      *Create an efficient design matrix based on a list of aspect models*

---

**Description**

Creates a unique design matrix based on model specification for each of the different potentially spatially varying aspects.

**Usage**

```
getDesignMatrix(model.list, data)
```

**Arguments**

`model.list`      (list) a list of formulas, one for each source of nonstationarity, specifying the models.  
`data`            (data.frame) a data.frame.

**Value**

(list) a list with two elements: a design matrix of dimension (n x p), and a par.pos object, indexing columns of the design matrix to each of the spatially-varying functions.

**Author(s)**

Federico Blasi

---

getEstims            *Retrieve estimates from a fitted coco object*

---

**Description**

Retrieve estimates from a fitted coco object.

**Usage**

```
getEstims(coco.object)
```

**Arguments**

`coco.object`      (S4) a fitted coco S4 object.

**Value**

(list) a list with the estimates parameters for the different aspects

**Author(s)**

Federico Blasi

---

|            |                   |
|------------|-------------------|
| getHessian | <i>getHessian</i> |
|------------|-------------------|

---

**Description**

numerically approximate the Hessian. Hessians of parameters based on "pml" are based on full likelihoods.

**Usage**

```
getHessian(coco.object, ncores = parallel::detectCores() - 1,  
eps = .Machine$double.eps^(1/4))
```

**Arguments**

|             |   |
|-------------|---|
| coco.object | (S4) a fitted coco object.                          |
| ncores      | (integer) number of cores used for the computation. |
| eps         | (numeric) ...                                       |

**Value**

(numeric matrix) a symmetric matrix p x p of the approximated (observed) Hessian

**Author(s)**

Federico Blasi

---

|           |   |
|-----------|---|
| getLoglik | <i>Retrieve the loglikelihood value</i> |
|-----------|---|

---

**Description**

Retrieve the loglikelihood value from a fitted coco object.

**Usage**

```
getLoglik(coco.object)
```

**Arguments**

|             |                               |
|-------------|-------------------------------|
| coco.object | (S4) a fitted coco S4 object. |
|-------------|-------------------------------|

**Value**

(numeric) wrap for value from a OptimParallel object

**Author(s)**

Federico Blasi

---

getLogScore

*Based on a set of predictions computes the Log-Score*

---

**Description**

Computes the Log-Score [1].

**Usage**

```
getLogScore(z.pred, mean.pred, sd.pred)
```

**Arguments**

z.pred (numeric vector).

mean.pred (numeric vector).

sd.pred (numeric vector).

**Value**

(numeric vector) retrieves Log-Score.

**Author(s)**

Federico Blasi

**References**

[1] Gneiting, Tilmann, and Adrian E. Raftery. "Strictly proper scoring rules, prediction, and estimation." *Journal of the American statistical Association* 102.477 (2007): 359-378.

---

|               |  |
|---------------|--|
| getModellists | <i>Builds the necessary input for building covariance matrices</i> |
|---------------|--|

---

**Description**

Returns a list of parameter vectors for each of the aspects.

**Usage**

```
getModellists(theta, par.pos, type = 'diff')
```

**Arguments**

|         |   |
|---------|---|
| theta   | (numeric vector) a vector of length p, where p is the number of parameters for each of the models.  |
| par.pos | (list) a list detailing in which position of each aspect the elements of theta should be placed. Expected to be par.pos output of <a href="#">getDesignMatrix</a> . |
| type    | (character) whether parameters are related to a classical parameterization ('classic') or a difference parameterization 'diff'. Default set to 'diff'.              |

**Value**

(list) a list of different spatial aspects and mean required for the cov.rns functions

**Author(s)**

Federico Blasi

---

|            |  |
|------------|--|
| getModHess | <i>Retrieves the modified inverse of the hessian</i> |
|------------|--|

---

**Description**

Based on the inverse of the Hessian (based on the difference parameterization for the std.dev and scale parameters), retrieves the modified inverse of the hessian (i.e. std.dev and scale).

**Usage**

```
getModHess(coco.object, inv.hess)
```

**Arguments**

|             |                                  |
|-------------|----------------------------------|
| coco.object | (S4) a fitted coco S4 object.    |
| inv.hess    | (matrix) Inverse of the Hessian. |

**Value**

(numeric matrix) the modified inverse of the hessian matrix

**Author(s)**

Federico Blasi

---

GetNeg2loglikelihood    *GetNeg2loglikelihood*

---

**Description**

compute the negative 2 log likelihood based on theta

**Usage**

```
GetNeg2loglikelihood(theta, par.pos, locs, x_covariates,  
smooth.limits, z, n, lambda, safe = TRUE)
```

**Arguments**

|               |  |
|---------------|--|
| theta         | (numeric vector) a vector with parameters values.  |
| par.pos       | (list) par.pos list.   |
| locs          | (matrix) spatial location matrix.  |
| x_covariates  | (data.frame) design matrix.  |
| smooth.limits | (numeric vector) smooth.limits.  |
| z             | (numeric vector) a vector of observed values.  |
| n             | (integer) dim(z)[1].   |
| lambda        | (numeric) regularization parameter.  |
| safe          | (TRUE/FALSE) if TRUE returns a large pre-defined value under Cholesky errors.<br>Default TRUE. |

**Value**

value

**Author(s)**

Federico Blasi

---

GetNeg2loglikelihoodProfile  
*GetNeg2loglikelihoodProfile*

---

**Description**

compute the negative 2 log likelihood based on theta

**Usage**

```
GetNeg2loglikelihoodProfile(theta, par.pos, locs, x_covariates,  
smooth.limits, z, n, x_betas, lambda, safe = TRUE)
```

**Arguments**

|               |  |
|---------------|--|
| theta         | (numeric vector) a vector with parameters values.  |
| par.pos       | (list) par.pos list.   |
| locs          | (matrix) spatial location matrix.  |
| x_covariates  | (data.frame) design matrix.  |
| smooth.limits | (numeric vector) smooth.limits.  |
| z             | (numeric vector) a vector of observed values.  |
| n             | (integer) dim(z)[1].   |
| x_betas       | (matrix) or (data.frame) design matrix for the spatial mean.                                   |
| lambda        | (numeric) regularization parameter.  |
| safe          | (TRUE/FALSE) if TRUE returns a large pre-defined value under Cholesky errors.<br>Default TRUE. |

**Value**

value

**Author(s)**

Federico Blasi

---

GetNeg2loglikelihoodREML

*GetNeg2loglikelihoodREML*

---

### Description

compute the negative 2 log REML likelihood based on theta

### Usage

```
GetNeg2loglikelihoodREML(theta, par.pos, locs, x_covariates, x_betas,  
smooth.limits, z, n, lambda, safe = TRUE)
```

### Arguments

|               |  |
|---------------|--|
| theta         | (numeric vector) a vector with parameters values.  |
| par.pos       | (list) par.pos list.   |
| locs          | (matrix) spatial location matrix.  |
| x_covariates  | (data.frame) design matrix.  |
| x_betas       | (matrix) or (data.frame) design matrix for the spatial mean.                                   |
| smooth.limits | (numeric vector) smooth.limits.  |
| z             | (numeric vector) a vector of contrasts.  |
| n             | (integer) dim(z)[1].   |
| lambda        | (numeric) regularization parameter.  |
| safe          | (TRUE/FALSE) if TRUE returns a large pre-defined value under Cholesky errors.<br>Default TRUE. |

### Value

value

### Author(s)

Federico Blasi

---

GetNeg2loglikelihoodTaper  
*GetNeg2loglikelihoodTaper*

---

### Description

compute the negative 2 log likelihood based on theta

### Usage

```
GetNeg2loglikelihoodTaper(theta, par.pos, ref_taper, locs,  
x_covariates, smooth.limits, cholS, z, n, lambda, safe = TRUE)
```

### Arguments

|               |  |
|---------------|--|
| theta         | (numeric vector) a vector with parameters values.  |
| par.pos       | (list) par.pos list from <a href="#">getDesignMatrix</a> .                                     |
| ref_taper     | (S4) spam object based on a compact-supported covariance function.                             |
| locs          | (matrix) spatial location matrix.  |
| x_covariates  | (data.frame) design matrix.  |
| smooth.limits | (numeric vector) smooth.limits.  |
| cholS         | (S4) Cholesky object from spam.  |
| z             | (numeric vector) a vector of observed values.  |
| n             | (numeric) dim(z)[1].   |
| lambda        | (numeric) regularization parameter.  |
| safe          | (TRUE/FALSE) if TRUE returns a large pre-defined value under Cholesky errors.<br>Default TRUE. |

### Value

value

### Author(s)

Federico Blasi

---

GetNeg2loglikelihoodTaperProfile  
*GetNeg2loglikelihoodTaperProfile*

---

**Description**

compute the negative 2 log likelihood based on theta

**Usage**

```
GetNeg2loglikelihoodTaperProfile(theta, par.pos, ref_taper,  
locs, x_covariates, smooth.limits, cholS, z, n, lambda, safe = TRUE)
```

**Arguments**

|               |  |
|---------------|--|
| theta         | (numeric vector) a vector with parameters values.  |
| par.pos       | (list) par.pos list.   |
| ref_taper     | (S4) spam object based on a taper based covariance function.                                   |
| locs          | (matrix) spatial location matrix.  |
| x_covariates  | (data.frame) design matrix.  |
| smooth.limits | (numeric vector) smooth.limits.  |
| cholS         | (S4) Cholesky object from spam.  |
| z             | (numeric vector) a vector of observed values.  |
| n             | (integer) dim(z)[1].   |
| lambda        | (numeric) regularization parameter.  |
| safe          | (TRUE/FALSE) if TRUE returns a large pre-defined value under Cholesky errors.<br>Default TRUE. |

**Value**

(numeric)

**Author(s)**

Federico Blasi

---

|          |   |
|----------|---|
| getScale | <i>Fast and simple standardization for the design matrix.</i> |
|----------|---|

---

**Description**

Centers and scale the design matrix.

**Usage**

```
getScale(x, mean.vector = NULL, sd.vector = NULL)
```

**Arguments**

|             |  |
|-------------|--|
| x           | (S4) or (matrix) a coco object, or a n x p matrix with covariate information to introduce, where the first column is a column of ones. |
| mean.vector | (numeric vector) if provided, it centers covariates based on this information.   |
| sd.vector   | (numeric vector) if provided, it scales covariates based on this information.  |

**Value**

(list) a list with a scaled design matrix of dimension n x (p+1), and a set of mean and sd vectors employed to scale the matrix

**Author(s)**

Federico Blasi

---

|                |   |
|----------------|---|
| getSpatEffects | <i>Evaluates the spatially-varying functions from a coco object at locs</i> |
|----------------|---|

---

**Description**

Evaluates the spatially-varying functions of the nonstationary spatial structure.

**Usage**

```
getSpatEffects(coco.object)
```

**Arguments**

|             |                               |
|-------------|-------------------------------|
| coco.object | (S4) a fitted coco S4 object. |
|-------------|-------------------------------|

**Value**

(list) a list with the different estimated surfaces.

**Author(s)**

Federico Blasi

---

|             |  |
|-------------|--|
| getSpatMean | <i>Computes the spatial mean of a (fitted) coco object</i> |
|-------------|--|

---

**Description**

Computes the spatial mean of the (fitted) coco object.

**Usage**

```
getSpatMean(coco.object)
```

**Arguments**

coco.object (S4) a fitted coco S4 object.

**Value**

(numeric vector) a vector with the adjusted trend.

**Author(s)**

Federico Blasi

---

|       |                       |
|-------|-----------------------|
| holes | <i>Holes Data Set</i> |
|-------|-----------------------|

---

**Description**

The synthetic "holes" provides a set of training and test data.frame of a Gaussian process realization with a (inherently dense) nonstationary covariance function. Four holes are present in the training dataset, and the task is to predict them.

**Usage**

```
holes
```

**Format**

A list with training and test data.frame with rows and variables:

**x** first spatial coordinate

**y** second spatial coordinate

**cox\_x** first spatial characteristic

**cov\_y** second spatial characteristic

**z** response variable

**Source**

Source of the data

**Examples**

```
data(holes)
```

---

holes\_bm

*Holes with trend + multiple realizations Data Set*

---

**Description**

The synthetic "holes\_bm" provides a set of training and test data.frame of a Gaussian process realization with a (inherently dense) nonstationary covariance function. Four holes are present in the training dataset, and the task is to predict them. This version provides ten independent realizations of the process, as well as considers a spatial mean effect.

**Usage**

```
holes_bm
```

**Format**

A list with training, training.z, test, and test.z data.frames with rows and variables:

**x** first spatial coordinate

**y** second spatial coordinate

**cox\_x** first spatial characteristic

**cov\_y** second spatial characteristic

**cov\_z** third spatial characteristic

**z.i** i-th response variable

**Source**

Source of the data

**Examples**

```
data(holes_bm)
```

---

|            |   |
|------------|---|
| is.formula | <i>check whether an R object is a formula</i> |
|------------|---|

---

**Description**

check whether an R object is a formula

**Usage**

```
is.formula(x)
```

**Arguments**

x (ANY) an R object.

**Value**

TRUE/FALSE

**Author(s)**

Federico Blasi

---

|                          |                                     |
|--------------------------|-------------------------------------|
| plot,coco,missing-method | <i>Plot Method for coco objects</i> |
|--------------------------|-------------------------------------|

---

**Description**

This method plots objects of class coco.

**Usage**

```
## S4 method for signature 'coco,missing'
plot(x, y, type = NULL, index = NULL, factr = 0.1, ...)
```

**Arguments**

|       |   |
|-------|---|
| x     | (S4) A fitted object of class coco.   |
| y     | Not used.   |
| type  | (character or NULL) The type of plot. NULL or "ellipse" for drawing ellipse of the convolution kernels. |
| index | (integer vector) For plotting local correlation plots.  |
| factr | (numeric) Factor rate for size of ellipses.   |
| ...   | Additional arguments passed to <a href="#">quilt.plot</a> .   |

**Value**

Several plots are created.

**Author(s)**

Federico Blasi

---

`plotOptimInfo`      *Plot log info detailed*

---

**Description**

plot output of optim

**Usage**

```
plotOptimInfo(coco.object, ...)
```

**Arguments**

`coco.object`      an optimized `coco.object`  
`...`              arguments for `par()`

**Value**

Outputs a sequence of plots detailing parameters during the optimization routine

**Author(s)**

Federico Blasi

**See Also**

[cocoOptim\(\)](#)

---

stripes

*Stripes Data Set*

---

### Description

The synthetic "stripes" provides a set of training and test data.frame of a Gaussian process realization with a (inherently sparse) nonstationary covariance function. Several stripes are present in the training dataset, and the task is to predict them.

### Usage

```
stripes
```

### Format

A list with training and test data.frame with rows and variables:

**x** first spatial coordinate

**y** second spatial coordinate

**cox\_x** first spatial characteristic

**cov\_y** second spatial characteristic

**cov\_xy** third spatial characteristic

**z** response variable

### Source

Source of the data

### Examples

```
data(stripes)
```

---

summary

*Summary Method for Coco Class*

---

### Description

method summary for objects of class 'coco'.

### Usage

```
## S4 method for signature 'coco'  
summary(object, inv.hess = NULL)
```

**Arguments**

object (S4) An object of class 'coco'.  
inv.hess (numeric matrix or NULL) inverse of the approximated hessian matrix (getHessian)

**Value**

summary the coco object

**Author(s)**

Federico Blasi

# Index

## \* datasets

- holes, [32](#)
- holes\_bm, [33](#)
- stripes, [36](#)

coco, [3](#), [3](#), [7](#), [8](#), [10](#), [11](#)

coco(), [19](#), [21](#)

coco-class, [6](#)

cocons (cocons-package), [3](#)

cocons-package, [3](#)

cocoOptim, [6](#)

cocoOptim(), [35](#)

cocoPredict, [8](#)

cocoSim, [10](#)

cov\_rns, [12](#)

cov\_rns\_classic, [12](#)

cov\_rns\_pred, [13](#)

cov\_rns\_taper, [14](#)

cov\_rns\_taper\_pred, [14](#)

getAIC, [15](#)

getBIC, [16](#)

getBoundaries, [4](#), [7](#), [16](#)

getBoundariesV2, [17](#)

getBoundariesV3, [18](#)

getCIs, [19](#)

getCovMatrix, [19](#)

getCRPS, [20](#)

getDensityFromDelta, [21](#)

getDesignMatrix, [16](#), [22](#), [25](#), [29](#)

getEstims, [22](#)

getHessian, [19](#), [23](#)

getLoglik, [23](#)

getLogScore, [24](#)

getModelLists, [25](#)

getModHess, [25](#)

GetNeg2loglikelihood, [26](#)

GetNeg2loglikelihoodProfile, [27](#)

GetNeg2loglikelihoodREML, [28](#)

GetNeg2loglikelihoodTaper, [29](#)

GetNeg2loglikelihoodTaperProfile, [30](#)

getScale, [31](#)

getSpatEffects, [31](#)

getSpatMean, [32](#)

holes, [32](#)

holes\_bm, [33](#)

is.formula, [34](#)

optimParallel, [4](#), [7](#)

plot, coco, missing-method, [34](#)

plot, coco-method

(plot, coco, missing-method), [34](#)

plotOptimInfo, [35](#)

quilt.plot, [34](#)

spam::cov.wend1(), [5](#)

stripes, [36](#)

summary, [36](#)

summary, coco-method (summary), [36](#)