

Package ‘FuzzyStatTra’

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Type Package

Title Statistical Methods for Trapezoidal Fuzzy Numbers

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Description The aim of the package is to provide some basic functions for doing statistics with trapezoidal fuzzy numbers. In particular, the package contains several functions for simulating trapezoidal fuzzy numbers, as well as for calculating some central tendency measures (mean and two types of median), some scale measures (variance, ADD, MDD, Sn, Qn, Tn and some M-estimators) and one diversity index and one inequality index. Moreover, functions for calculating the 1-norm distance, the mid/spr distance and the (phi,theta)-wabl/ldev/rdev distance between fuzzy numbers are included, and a function to calculate the value phi-wabl given a sample of trapezoidal fuzzy numbers.

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R topics documented:

FuzzyStatTra-package	2
ADD	4
checking	6
checkingTra	7
Dthetaphi	9
DthetaphiTra	10
Dwablphi	12

DwablphiTra	13
filterNA	15
GSI	16
HyperI	17
M.estimate	18
M1	20
M2	21
M3	22
MDD	23
Mean	25
MedianInorm	26
Medianwabl	27
Qn	29
Rho1	30
Rho1Tra	31
S1	33
SimulCASE1	34
SimulCASE2	35
SimulCASE3	36
SimulCASE4	37
SimulFRSTra	38
Sn	39
Tn	40
TransfTra	42
Var	43
Wablphi	44
Index	46

FuzzyStatTra-package *Statistical Methods for Trapezoidal Fuzzy Numbers*

Description

The aim of the package is to provide some basic functions for doing statistics with trapezoidal fuzzy numbers.

Details

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Type: Package
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The aim of the package is to provide some basic functions for doing statistics with trapezoidal fuzzy numbers. In particular, the package contains several functions for simulating trapezoidal fuzzy numbers, as well as for calculating some central tendency measures (mean and two types of median), some scale measures (variance, ADD, MDD, Sn, Qn, Tn and some M-estimators) and one diversity index and one inequality index. Moreover, functions for calculating the 1-norm distance, the mid/spr distance and the (φ, θ) -wabl/ldev/rdev distance between fuzzy numbers are included, and a function to calculate the value φ -wabl given a sample of trapezoidal fuzzy numbers.

Author(s)

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References

- [1] Blanco-Fernandez, A.; Casals, R.M.; Colubi, A.; Corral, N.; Garcia-Barzana, M.; Gil, M.A.; Gonzalez-Rodriguez, G.; Lopez, M.T.; Lubiano, M.A.; Montenegro, M.; Ramos-Guajardo, A.B.; de la Rosa de Saa, S.; Sinova, B.: *Random fuzzy sets: A mathematical tool to develop statistical fuzzy data analysis*, Iranian Journal on Fuzzy Systems 10(2), pp. 1-28 (2013)
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See Also

<http://bellman.ciencias.uniovi.es/SMIRE/>

ADD	<i>Average Distance Deviation of a trapezoidal fuzzy sample with respect to a fuzzy number</i>
-----	--

Description

This function calculates the scale measure Average Distance Deviation (ADD) for a matrix of trapezoidal fuzzy numbers F with respect to a fuzzy number U . The employed metric in the calculation can be the 1-norm distance, the mid/spr distance or the (φ, θ) -wabl/ldev/rdev distance. The function first checks if the input matrix F is given in the correct form (tested by checkingTra) and also the input fuzzy number U (tested by checking or checkingTra).

Usage

ADD(F , U , type, a = 1, b = 1, theta = 1/3)

Arguments

F	matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta, \text{inf}1, \text{sup}1, \text{sup}\theta$. The function implicitly checks if the matrix is in the correct form (tested by checkingTra).
U	can be a matrix of dimension 1×4 (trapezoidal fuzzy number) or an array of dimension $n1 \times 3 \times 1$ (general fuzzy number), where $n1$ is the number of considered α -levels and 3 the number of columns of the array: the first column will be the α -levels, the second one their infimum values and the third one their supremum values. The function implicitly checks if the fuzzy number U is in the correct form (tested by checking if U is an array and tested by checkingTra if U is a matrix).
type	number 1, 2 or 3: if type==1, the 1-norm distance will be considered in the calculation of the measure ADD. If type==2, the mid/spr distance will be considered. By contrast, if type==3, the (φ, θ) -wabl/ldev/rdev distance will be used.
a	number >0, by default a=1. It is the first parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$ in the mid/spr distance or in the (φ, θ) -wabl/ldev/rdev distance.
b	number >0, by default b=1. It is the second parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$ in the mid/spr distance or in the (φ, θ) -wabl/ldev/rdev distance.
theta	number >0, by default theta=1/3. It is the weight of the spread in the mid/spr distance and the weight of the ldev and rdev in the (φ, θ) -wabl/ldev/rdev distance.

Details

See examples

Value

The function returns the scale measure ADD, which is a real number.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

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References

[1] De la Rosa de Saa, S.; Lubiano M.A.; Sinova, B.; Filzmoser, P.: *Robust scale estimators for fuzzy data*, Advances in Data Analysis and Classification, pp. 1-28 (2015)

See Also

[checkingTra](#), [checking](#), [TransfTra](#), [Rho1Tra](#), [Rho1](#), [DthetaphiTra](#), [Dthetaphi](#), [DwablphiTra](#), [Dwablphi](#)

Examples

```
# Example 1:
F=SimulCASE1(10)
U=Mean(F)
ADD(F,U,1)

# Example 2:
F=SimulCASE1(100)
U=Median1norm(F)
ADD(F,U,2,2,1,1)

# Example 3:
F=SimulCASE1(100)
U=matrix(c(1,2,3,2),nrow=1)
ADD(F,U,1)

# Example 4:
F=matrix(1:4,nrow=2)
U=matrix(1:4,nrow=1)
ADD(F,U,3,1,1,1)
```

checking

Checking correct data format (as array)

Description

The function checks if the input data are given in the correct form of an array of dimension $n1 \times 3 \times n$ containing n fuzzy numbers characterized by means of $n1$ α -levels each. The following conditions have to be fulfilled: (1) the number of columns of the array must be 3 (the first column will be the α -levels, the second one their infimum values and the third one their supremum values), (2) all the fuzzy numbers must have the same column of α -levels, (3) the minimum α -level should be 0 y the maximum 1, (4) the α -levels have to increase from 0 to 1, (5) the infimum values have to be non-decreasing, (6) the supremum values have to be non-creasing, (7) the infimum value has to be smaller or equal than the supremum value for each α -level. This function is used internally in some of the other functions to do a preliminary checking if the input data are in the correct form.

Usage

checking(R)

Arguments

R can be any array.

Details

See examples

Value

The function returns the value 1 if the input fulfills all conditions, if not, the value 0 is returned.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

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See Also

[checkingTra](#), [filterNA](#)

Examples

```

# Example 1:
F=SimulCASE1(10)
R=TransfTra(F)
c=checking(R)
c

# Example 2:
R=array(c(1:10),dim=c(2,1,2))
c=checking(R)
c

# Example 3:
R=array(c(1:10),dim=c(2,3,2))
c=checking(R)
c

# Example 4:
R=array(c(1,2,3,4,5,6,1,2,4,5,6,7),dim=c(2,3,2))
c=checking(R)
c

# Example 5:
R=array(c(0,0,1,2,3,4,5,0,1,0,0,1,7,8,9,19,30,3),dim=c(3,3,2))
c=checking(R)
c

# Example 6:
R=array(c(0,0.5,1,2,3,4,5,0,1,0,0.5,1,7,8,7,19,30,3),dim=c(3,3,2))
c=checking(R)
c

# Example 7:
R=array(c(0,0.5,1,2,3,4,5,0,1,0,0.5,1,7,8,9,19,30,3),dim=c(3,3,2))
c=checking(R)
c

# Example 8:
R=array(c(0,0.5,1,2,3,4,6,5,4,0,0.5,1,7,8,9,19,10,2),dim=c(3,3,2))
c=checking(R)
c

```

checkingTra

Checking correct data format (as matrix)

Description

The function checks if the input data are given in the correct form of a matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}0$, $\text{inf}1$, $\text{sup}1$, $\text{sup}0$ each. The following conditions have to be fulfilled: (1) the number of columns of the matrix must

be 4 (the four values characterizing each trapezoidal fuzzy number), (2) the four values of each trapezoidal number have to be non-decreasing. This function is used internally in almost all the other functions to do a preliminary checking if the input data are in the correct form.

Usage

```
checkingTra(F)
```

Arguments

F can be any matrix.

Details

See examples

Value

The function returns the value 1 if the input fulfills all conditions, if not, the value 0 is returned.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

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See Also

[checking](#), [filterNA](#)

Examples

```
# Example 1:
F=matrix(c(1,2,3,4),nrow=1)
c=checkingTra(F)
c

# Example 2:
F=matrix(c(1,2,3,4),nrow=2)
c=checkingTra(F)
c

# Example 3:
F=matrix(c(1,2,1,4),nrow=1)
c=checkingTra(F)
c
```


Dthetaphi

*Mid/spr distance between fuzzy numbers***Description**

This function calculates the mid/spr distance between the fuzzy numbers contained in two arrays, which should be given in the desired format. For this, the function first checks if the input arrays R and S are in the correct form (tested by checking) and if the α -levels of all fuzzy numbers coincide.

Usage

Dthetaphi(R, S, a = 1, b = 1, theta = 1/3)

Arguments

R	array of dimension $n1 \times 3 \times r$ containing r fuzzy numbers characterized by means of $n1$ α -levels each. The function first calls checking to check if the array R has the correct format. Moreover, the α -levels of the array R should coincide with the ones of the array S (the function checks this condition).
S	array of dimension $n1 \times 3 \times s$ containing s fuzzy numbers characterized by means of $n1$ α -levels each. The function first calls checking to check if the array S has the correct format. Moreover, the α -levels of the array S should coincide with the ones of the array R (the function checks this condition).
a	number >0 , by default $a=1$. It is the first parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$.
b	number >0 , by default $b=1$. It is the second parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$.
theta	number >0 , by default $\theta=1/3$. It is the weight of the spread in the mid/spr distance.

Details

See examples

Value

The function returns a matrix of dimension $r \times s$ containing the mid/spr distances between the fuzzy numbers of the array R and the fuzzy numbers of the array S .

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

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References

[1] Blanco-Fernandez, A.; Casals, R.M.; Colubi, A.; Corral, N.; Garcia-Barzana, M.; Gil, M.A.; Gonzalez-Rodriguez, G.; Lopez, M.T.; Lubiano, M.A.; Montenegro, M.; Ramos-Guajardo, A.B.; de la Rosa de Saa, S.; Sinova, B.: *Random fuzzy sets: A mathematical tool to develop statistical fuzzy data analysis*, Iranian Journal on Fuzzy Systems 10(2), pp. 1-28 (2013)

See Also

[checking](#), [DthetaphiTra](#)

Examples

```
# Example 1:
F=SimulCASE1(10)
S=SimulCASE1(20)
F=TransfTra(F)
S=TransfTra(S)
Dthetaphi(F,S,1,5,1)

# Example 2:
F=SimulCASE1(10)
S=SimulCASE1(10)
Dthetaphi(F,S,2,1,1/3)

# Example 3:
F=SimulCASE1(10)
S=SimulCASE1(10)
F=TransfTra(F)
S=TransfTra(S,50)
Dthetaphi(F,S,2,1,1)
```

DthetaphiTra

Mid/spr distance between trapezoidal fuzzy numbers

Description

This function calculates the mid/spr distance between the trapezoidal fuzzy numbers contained in two matrixes, which should be given in the desired format. For this, the function first checks if the input matrixes R and S are in the correct form (tested by `checkingTra`).

Usage

```
DthetaphiTra(R, S, a = 1, b = 1, theta = 1/3)
```

Arguments

R	matrix of dimension $r \times 4$ containing r trapezoidal fuzzy numbers characterized by their four values $\text{inf}0, \text{inf}1, \text{sup}1, \text{sup}0$. The function first calls <code>checkingTra</code> to check if the matrix R has the correct format.
S	matrix of dimension $s \times 4$ containing s trapezoidal fuzzy numbers characterized by their four values $\text{inf}0, \text{inf}1, \text{sup}1, \text{sup}0$. The function first calls <code>checkingTra</code> to check if the matrix S has the correct format.
a	number >0 , by default $a=1$. It is the first parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$.
b	number >0 , by default $b=1$. It is the second parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$.
theta	number >0 , by default $\text{theta}=1/3$. It is the weight of the spread in the mid/spr distance.

Details

See examples

Value

The function returns a matrix of dimension $r \times s$ containing the mid/spr distances between the trapezoidal fuzzy numbers of the matrix R and the trapezoidal fuzzy numbers of the matrix S.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

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References

[1] Lubiano, M.A.; Montenegro, M.; Sinova, B.; De la Rosa de Saa, S.; Gil, M.A.: *Hypothesis testing for means in connection with fuzzy rating scale-based data: algorithms and applications*, European Journal of Operational Research 251, pp. 918-929 (2016)

See Also

[checkingTra](#), [Dthetaphi](#)

Examples

```
# Example 1:
F=SimulCASE1(6)
S=SimulCASE1(8)
DthetaphiTra(F,S)
```

```
# Example 2:
F=matrix(c(1,1,0,2,3,4,5,6),nrow=2)
S=SimulCASE1(8)
DthetaphiTra(F,S,1,1,1)
```

Dwablphi

(φ, θ)-wabl/ldev/rdev distance between fuzzy numbers

Description

This function calculates the (φ, θ) -wabl/ldev/rdev distance between the fuzzy numbers contained in two arrays, which should be given in the desired format. For this, the function first checks if the input arrays R and S are in the correct form (tested by checking) and if the α -levels of all fuzzy numbers coincide.

Usage

```
Dwablphi(R, S, a = 1, b = 1, theta = 1)
```

Arguments

- | | |
|-------|---|
| R | array of dimension $n1 \times 3 \times r$ containing r fuzzy numbers characterized by means of $n1$ α -levels each. The function first calls checking to check if the array R has the correct format. Moreover, the α -levels of the array R should coincide with the ones of the array S (the function checks this condition). |
| S | array of dimension $n1 \times 3 \times s$ containing s fuzzy numbers characterized by means of $n1$ α -levels each. The function first calls checking to check if the array S has the correct format. Moreover, the α -levels of the array S should coincide with the ones of the array R (the function checks this condition). |
| a | number >0 , by default $a=1$. It is the first parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$. |
| b | number >0 , by default $b=1$. It is the second parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$. |
| theta | number >0 , by default $\theta=1$. It is the weight of the ldev and rdev in the (φ, θ) -wabl/ldev/rdev distance. |

Details

See examples

Value

The function returns a matrix of dimension $r \times s$ containing the (φ, θ) -wabl/ldev/rdev distances between the fuzzy numbers of the array R and the fuzzy numbers of the array S .

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

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References

- [1] Sinova, B.; de la Rosa de Saa, S.; Gil, M.A.: *A generalized L1-type metric between fuzzy numbers for an approach to central tendency of fuzzy data*, Information Sciences 242, pp. 22-34 (2013)
- [2] Sinova, B.; Gil, M.A.; Van Aelst, S.: *M-estimates of location for the robust central tendency of fuzzy data*, IEEE Transactions on Fuzzy Systems 24(4), pp. 945-956 (2016)

See Also

[checking](#), [DwablphiTra](#), [Wablphi](#)

Examples

```
# Example 1:
F=SimulCASE1(3)
S=SimulCASE1(4)
F=TransfTra(F)
S=TransfTra(S)
Dwablphi(F,S,2,1,1)

# Example 2:
F=SimulCASE1(10)
S=SimulCASE1(10)
Dwablphi(F,S)

# Example 3:
F=SimulCASE1(10)
S=SimulCASE1(10)
F=TransfTra(F)
S=TransfTra(S,50)
Dwablphi(F,S,2,1,1)
```

DwablphiTra

 (φ, θ) -wabl/ldev/rdev distance between trapezoidal fuzzy numbers

Description

This function calculates the (φ, θ) -wabl/ldev/rdev distance between the trapezoidal fuzzy numbers contained in two matrixes, which should be given in the desired format. For this, the function first checks if the input matrixes R and S are in the correct form (tested by [checkingTra](#)).

Usage

DwablphiTra(R, S, a = 1, b = 1, theta = 1)

Arguments

R	matrix of dimension $r \times 4$ containing r trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta$, $\text{inf}1$, $\text{sup}1$, $\text{sup}\theta$. The function first calls <code>checkingTra</code> to check if the matrix R has the correct format.
S	matrix of dimension $s \times 4$ containing s trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta$, $\text{inf}1$, $\text{sup}1$, $\text{sup}\theta$. The function first calls <code>checkingTra</code> to check if the matrix S has the correct format.
a	number >0 , by default $a=1$. It is the first parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$.
b	number >0 , by default $b=1$. It is the second parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$.
theta	number >0 , by default $\text{theta}=1$. It is the weight of the ldev and rdev in the (φ, θ) - wabl/ldev/rdev distance.

Details

See examples

Value

The function returns a matrix of dimension $r \times s$ containing the (φ, θ) - wabl/ldev/rdev distances between the trapezoidal fuzzy numbers of the matrix R and the trapezoidal fuzzy numbers of the matrix S.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

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References

- [1] Sinova, B.; de la Rosa de Saa, S.; Gil, M.A.: *A generalized L1-type metric between fuzzy numbers for an approach to central tendency of fuzzy data*, Information Sciences 242, pp. 22-34 (2013)
- [2] Sinova, B.; Gil, M.A.; Van Aelst, S.: *M-estimates of location for the robust central tendency of fuzzy data*, IEEE Transactions on Fuzzy Systems 24(4), pp. 945-956 (2016)

See Also

[checkingTra](#), [Dwablphi](#), [Wablphi](#)

Examples

```
# Example 1:
F=SimulCASE1(10)
S=SimulCASE1(20)
DwablphiTra(F,S,5,1,1)

# Example 2:
F=matrix(c(1,1,0,2,3,4,5,6),nrow=2)
S=SimulCASE1(8)
DwablphiTra(F,S)
```

filterNA	<i>Deleting missing values</i>
----------	--------------------------------

Description

Given any matrix, this function deletes those rows with missing values.

Usage

```
filterNA(F)
```

Arguments

F can be any matrix.

Details

See examples

Value

The function returns a list with two components: the first one is a matrix identical to the input matrix F but without the rows containing missing values, and the second component is the number of rows of the input matrix without missing values.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

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See Also

[checkingTra](#), [checking](#)

Examples

```
# Example 1:
F=matrix(c(1,2,3,NA,5,4,7,2),nrow=2)
filterNA(F)

# Example 2:
F=matrix(c(1,2,3,NA,5,4,7,2,1,2,3,4),nrow=3)
filterNA(F)

# Example 3:
data(M2)
filterNA(M2)
```

GSI

Gini-Simpson diversity index of a trapezoidal fuzzy sample

Description

This function calculates the Gini-Simpson diversity index for a sample of trapezoidal fuzzy numbers contained in a matrix F. The function first checks if the input matrix F is given in the correct form (tested by checkingTra).

Usage

```
GSI(F)
```

Arguments

F matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta, \text{inf}1, \text{sup}1, \text{sup}\theta$. The function implicitly checks if the matrix is in the correct form (tested by checkingTra).

Details

See examples

Value

The function returns the Gini-Simpson diversity index, which is a real number.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

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References

[1] De la Rosa de Saa, S.; Gil, M.A.; Gonzalez-Rodriguez, G.; Lopez, M.T.; Lubiano M.A.: *Fuzzy rating scale-based questionnaires and their statistical analysis*, IEEE Trans. Fuzzy Syst. 23(1), pp. 111-126 (2015)

See Also

[checkingTra](#)

Examples

```
# Example 1:
F=SimulCASE1(50)
GSI(F)

# Example 2:
F=matrix(c(1,0,2,3),nrow=1)
GSI(F)
```

HyperI

Hyperbolic inequality index of a trapezoidal positive fuzzy sample

Description

This function calculates the hyperbolic inequality index for a sample of trapezoidal positive fuzzy numbers contained in a matrix F. The function first checks if the input matrix F is given in the correct form (tested by `checkingTra`).

Usage

```
HyperI(F, c = 0)
```

Arguments

F	matrix of dimension $n \times 4$ containing n trapezoidal positive fuzzy numbers characterized by their four values $\text{inf}0, \text{inf}1, \text{sup}1, \text{sup}0$. The function implicitly checks if the matrix is in the correct form (tested by <code>checkingTra</code>).
c	number in $[0,0.5]$. The $c*100\%$ trimmed mean will be used in the calculation of the hyperbolic inequality index.

Details

See examples

Value

The function returns the hyperbolic inequality index, which is a real number.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

References

- [1] Lubiano, M.A.; Gil, M.A.: *f-Inequality indices for fuzzy random variables*, in Statistical Modeling, Analysis and Management of Fuzzy Data (Bertoluzza, C., Gil, M.A., Ralescu, D.A., Eds.), Physica-Verlag, pp. 43-63 (2002)
- [2] De la Rosa de Saa, S.; Gil, M.A.; Gonzalez-Rodriguez, G.; Lopez, M.T.; Lubiano M.A.: *Fuzzy rating scale-based questionnaires and their statistical analysis*, IEEE Trans. Fuzzy Syst. 23(1), pp. 111-126 (2015)

See Also

[checkingTra](#)

Examples

```
# Example 1:
F=SimulFRSTra(100,6,0.05,0.35,0.6,2,1)
HyperI(F)

# Example 2:
F=SimulCASE2(10)
HyperI(F,0.5)
```

M.estimate

M-estimator of scale of a trapezoidal fuzzy sample

Description

This function calculates the M-estimator of scale with loss function given in M for a matrix of trapezoidal fuzzy numbers F. For computing the M-estimator, a method called “iterative reweighting” is used. The employed metric in the M-equation can be the 1-norm distance, the mid/spr distance or the (φ, θ) -wabl/ldev/rdev distance. The function first checks if the input matrix F is given in the correct form (tested by checkingTra).

Usage

```
M.estimate(F, M, est_initial, delta, epsilon, type, a = 1, b = 1, theta = 1/3)
```

Arguments

F	matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta, \text{inf}1, \text{sup}1, \text{sup}\theta$. The function implicitly checks if the matrix is in the correct form (tested by <code>checkingTra</code>).
M	name of the loss function. It can be “Huber”, “Tukey” or “Cauchy”.
est_initial	initial scale estimate.
delta	number in $(0,1)$. It is present in the M-equation.
epsilon	number >0 . It is the tolerance allowed in the algorithm.
type	number 1, 2 or 3: if <code>type==1</code> , the 1-norm distance will be considered in the calculation of the M-estimator. If <code>type==2</code> , the mid/spr distance will be considered. By contrast, if <code>type==3</code> , the (φ, θ) -wabl/ldev/rdev distance will be used.
a	number >0 , by default <code>a=1</code> . It is the first parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$ in the mid/spr distance or in the (φ, θ) -wabl/ldev/rdev distance.
b	number >0 , by default <code>b=1</code> . It is the second parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$ in the mid/spr distance or in the (φ, θ) -wabl/ldev/rdev distance.
theta	number >0 , by default <code>theta=1/3</code> . It is the weight of the spread in the mid/spr distance and the weight of the ldev and rdev in the (φ, θ) -wabl/ldev/rdev distance.

Details

See examples

Value

The function returns the value of the M-estimator of scale, which is a real number.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

See Also

[checkingTra](#), [Rho1Tra](#), [DthetaphiTra](#), [DwablphiTra](#)

Examples

```
# Example 1:  
F=SimulCASE1(100)  
U=Median1norm(F)  
est_initial=MDD(F,U,1)  
delta=0.5  
epsilon=10^(-5)  
M.estimate(F,"Huber",est_initial,delta,epsilon,1)
```

M1

M1 dataset

Description

M1 is a matrix of dimension 69 x 4 containing 69 trapezoidal fuzzy rating responses, each of which is characterized by its four values inf_0 , inf_1 , sup_1 , sup_0 . The data correspond to the well-known questionnaire TIMSS-PIRLS2011. This questionnaire was adapted to allow a double-type response, namely, the original Likert and a fuzzy rating scale-based (to simplify, trapezoidal). The questionnaire was conducted on 69 fourth grade students from Colegio San Ignacio (Oviedo-Asturias, Spain). Trapezoidal fuzzy rating responses to the Question M1 "I like mathematics" are collected in this dataset.

Usage

```
data("M1")
```

Format

A matrix of dimension 69 x 4 containing 69 trapezoidal fuzzy rating responses, each of which is characterized by its four values inf_0 , inf_1 , sup_1 , sup_0 .

Details

See examples

Source

The complete dataset can be found in <http://bellman.ciencias.uniovi.es/SMIRE/FuzzyRatingScaleQuestionnaire-SanIgnacio.html>

References

- [1] Gil, M.A.; Lubiano, M.A.; De la Rosa de Saa, S.; Sinova, B.: *Analyzing data from a fuzzy rating scale-based questionnaire. A case study*, *Psicothema* 27(2), pp. 182-191 (2015)
- [2] Lubiano, M.A.; De la Rosa de Saa, S.; Montenegro, M.; Sinova, B.; Gil, M.A.: *Descriptive analysis of responses to items in questionnaires. Why not a fuzzy rating scale?*, *Information Sciences* 360, pp. 131-148 (2016)

[3] Lubiano, M.A.; Montenegro, M.; Sinova, B.; De la Rosa de Saa, S.; Gil, M.A.: *Hypothesis testing for means in connection with fuzzy rating scale-based data: algorithms and applications*, European Journal of Operational Research 251, pp. 918-929 (2016)

Examples

```
data(M1)
filterNA(M1)
F=filterNA(M1)[[1]]
Medianwabl(F)
```

M2

M2 dataset

Description

M2 is a matrix of dimension 69 x 4 containing 69 trapezoidal fuzzy rating responses, each of which is characterized by its four values inf_0 , inf_1 , sup_1 , sup_0 . The data correspond to the well-known questionnaire TIMSS-PIRLS2011. This questionnaire was adapted to allow a double-type response, namely, the original Likert and a fuzzy rating scale-based (to simplify, trapezoidal). The questionnaire was conducted on 69 fourth grade students from Colegio San Ignacio (Oviedo-Asturias, Spain). Trapezoidal fuzzy rating responses to the Question M2 "My teacher is easy to understand" are collected in this dataset.

Usage

```
data("M2")
```

Format

A matrix of dimension 69 x 4 containing 69 trapezoidal fuzzy rating responses, each of which is characterized by its four values inf_0 , inf_1 , sup_1 , sup_0 .

Details

See examples

Source

The complete dataset can be found in <http://bellman.ciencias.uniovi.es/SMIRE/FuzzyRatingScaleQuestionnaire-SanIgnacio.html>

References

- [1] Gil, M.A.; Lubiano, M.A.; De la Rosa de Saa, S.; Sinova, B.: *Analyzing data from a fuzzy rating scale-based questionnaire. A case study*, *Psicothema* 27(2), pp. 182-191 (2015)
- [2] Lubiano, M.A.; De la Rosa de Saa, S.; Montenegro, M.; Sinova, B.; Gil, M.A.: *Descriptive analysis of responses to items in questionnaires. Why not a fuzzy rating scale?*, *Information Sciences* 360, pp. 131-148 (2016)
- [3] Lubiano, M.A.; Montenegro, M.; Sinova, B.; De la Rosa de Saa, S.; Gil, M.A.: *Hypothesis testing for means in connection with fuzzy rating scale-based data: algorithms and applications*, *European Journal of Operational Research* 251, pp. 918-929 (2016)

Examples

```
data(M2)
filterNA(M2)
F=filterNA(M2)[[1]]
Mean(F)
```

M3

M3 dataset

Description

M3 is a matrix of dimension 69 x 4 containing 69 trapezoidal fuzzy rating responses, each of which is characterized by its four values $\text{inf}\theta$, $\text{inf}1$, $\text{sup}1$, $\text{sup}\theta$. The data correspond to the well-known questionnaire TIMSS-PIRLS2011. This questionnaire was adapted to allow a double-type response, namely, the original Likert and a fuzzy rating scale-based (to simplify, trapezoidal). The questionnaire was conducted on 69 fourth grade students from Colegio San Ignacio (Oviedo-Asturias, Spain). Trapezoidal fuzzy rating responses to the Question M3 "Mathematics is harder for me than any other subject" are collected in this dataset.

Usage

```
data("M3")
```

Format

A matrix of dimension 69 x 4 containing 69 trapezoidal fuzzy rating responses, each of which is characterized by its four values $\text{inf}\theta$, $\text{inf}1$, $\text{sup}1$, $\text{sup}\theta$.

Details

See examples

Source

The complete dataset can be found in <http://bellman.ciencias.uniovi.es/SMIRE/FuzzyRatingScaleQuestionnaire-SanIgnacio.html>

References

- [1] Gil, M.A.; Lubiano, M.A.; De la Rosa de Saa, S.; Sinova, B.: *Analyzing data from a fuzzy rating scale-based questionnaire. A case study*, *Psicothema* 27(2), pp. 182-191 (2015)
- [2] Lubiano, M.A.; De la Rosa de Saa, S.; Montenegro, M.; Sinova, B.; Gil, M.A.: *Descriptive analysis of responses to items in questionnaires. Why not a fuzzy rating scale?*, *Information Sciences* 360, pp. 131-148 (2016)
- [3] Lubiano, M.A.; Montenegro, M.; Sinova, B.; De la Rosa de Saa, S.; Gil, M.A.: *Hypothesis testing for means in connection with fuzzy rating scale-based data: algorithms and applications*, *European Journal of Operational Research* 251, pp. 918-929 (2016)

Examples

```
data(M3)
filterNA(M3)
F=filterNA(M3)[[1]]
Median1norm(F)
```

MDD

Median Distance Deviation of a trapezoidal fuzzy sample with respect to a fuzzy number

Description

This function calculates the scale measure Median Distance Deviation (MDD) for a matrix of trapezoidal fuzzy numbers F with respect to a fuzzy number U . The employed metric in the calculation can be the 1-norm distance, the mid/spr distance or the (φ, θ) -wabl/ldev/rdev distance. The function first checks if the input matrix F is given in the correct form (tested by checkingTra) and also the input fuzzy number U (tested by checking or checkingTra).

Usage

```
MDD(F, U, type, a = 1, b = 1, theta = 1/3)
```

Arguments

- F matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta, \text{inf}1, \text{sup}1, \text{sup}\theta$. The function implicitly checks if the matrix is in the correct form (tested by checkingTra).
- U can be a matrix of dimension 1×4 (trapezoidal fuzzy number) or an array of dimension $n1 \times 3 \times 1$ (general fuzzy number), where $n1$ is the number of considered α -levels and 3 the number of columns of the array: the first column will be the α -levels, the second one their infimum values and the third one their supremum values. The function implicitly checks if the fuzzy number U is in the correct form (tested by checking if U is an array and tested by checkingTra if U is a matrix).

type	number 1, 2 or 3: if type==1, the 1-norm distance will be considered in the calculation of the measure MDD. If type==2, the mid/spr distance will be considered. By contrast, if type==3, the (φ, θ) -wabl/ldev/rdev distance will be used.
a	number >0, by default a=1. It is the first parameter of a beta distribution which corresponds to a weighting measure on [0,1] in the mid/spr distance or in the (φ, θ) -wabl/ldev/rdev distance.
b	number >0, by default b=1. It is the second parameter of a beta distribution which corresponds to a weighting measure on [0,1] in the mid/spr distance or in the (φ, θ) -wabl/ldev/rdev distance.
theta	number >0, by default theta=1/3. It is the weight of the spread in the mid/spr distance and the weight of the ldev and rdev in the (φ, θ) -wabl/ldev/rdev distance.

Details

See examples

Value

The function returns the scale measure MDD, which is a real number.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

References

[1] De la Rosa de Saa, S.; Lubiano M.A.; Sinova, B.; Filzmoser, P.: *Robust scale estimators for fuzzy data*, Advances in Data Analysis and Classification, pp. 1-28 (2015)

See Also

[checkingTra](#), [checking](#), [TransfTra](#), [Rho1Tra](#), [Rho1](#), [DthetaphiTra](#), [Dthetaphi](#), [DwablphiTra](#), [Dwablphi](#)

Examples

```
# Example 1:
F=SimulCASE3(10)
U=Mean(F)
MDD(F,U,3,1,2,1)

# Example 2:
F=SimulCASE2(10)
```



```

U=Median1norm(F)
MDD(F,U,2)

# Example 3:
F=SimulCASE1(100)
U=matrix(c(1,2,3,2),nrow=1)
MDD(F,U,1)

# Example 4:
F=SimulCASE1(100)
U=array(1:60,dim=c(10,2,3))
MDD(F,U,2,1,2,1)

```

Mean

Mean of a trapezoidal fuzzy sample

Description

Given a sample of trapezoidal fuzzy numbers contained in a matrix F , the function calculates the Aumann-type mean of these numbers (which is a trapezoidal fuzzy number too). The function first checks if the input matrix F is given in the correct form (tested by `checkingTra`).

Usage

```
Mean(F)
```

Arguments

F matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta, \text{inf}1, \text{sup}1, \text{sup}\theta$. The function implicitly checks if the matrix is in the correct form (tested by `checkingTra`).

Details

See examples

Value

The function returns the Aumann-type mean, given as a matrix of dimension 1×4 .

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

References

[1] Sinova, B.; De la Rosa de Saa, S.; Lubiano, M.A.; Gil, M.A.: *An overview on the statistical central tendency for fuzzy datasets*, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 23 (Suppl. 1), pp. 105-132 (2015)

See Also

[checkingTra](#)

Examples

```
# Example 1:
F=SimulCASE1(100)
Mean(F)

# Example 2:
F=matrix(c(1,0,2,3),nrow=1)
Mean(F)

# Example 3:
F=matrix(c(1,0,2,3),nrow=2)
Mean(F)
```

Median1norm

1-norm median of a trapezoidal fuzzy sample

Description

Given a sample of trapezoidal fuzzy numbers contained in a matrix F, the function calculates the 1-norm median of these numbers, characterized by means of n1 equidistant α -levels (by default n1=101), including always the 0 and 1 levels, with their infimum and supremum values. The function first checks if the input matrix F is given in the correct form (tested by checkingTra).

Usage

```
Median1norm(F, n1 = 101)
```

Arguments

F	matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}0, \text{inf}1, \text{sup}1, \text{sup}0$. The function implicitly checks if the matrix is in the correct form (tested by checkingTra).
n1	positive integer, by default n1=101. It indicates the number of desired α -levels for characterizing the 1-norm median.

Details

See examples

Value

The function returns the 1-norm median, given by an array of dimension $n1 \times 3 \times 1$ where $n1$ is the number of considered α -levels and 3 the number of columns of the array: the first column will be the α -levels, the second one their infimum values and the third one their supremum values.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

References

[1] Sinova, B.; Gil, M.A.; Colubi, A.; Van Aelst, S.: *The median of a random fuzzy number. The 1-norm distance approach*, Fuzzy Sets Syst. 200, pp. 99-115 (2012)

See Also

[checkingTra](#), [TransfTra](#), [Medianwabl](#)

Examples

```
# Example 1:
F=SimulCASE1(10)
Median1norm(F,200)

# Example 2:
F=matrix(c(1,0,2,3),nrow=1)
Median1norm(F)
```

Medianwabl

 φ -wabl/ldev/rdev median of a trapezoidal fuzzy sample

Description

Given a sample of trapezoidal fuzzy numbers contained in a matrix F , the function calculates the φ -wabl/ldev/rdev median of these numbers, characterized by means of $n1$ equidistant α -levels (by default $n1=101$), including always the 0 and 1 levels, with their infimum and supremum values. The function first checks if the input matrix F is given in the correct form (tested by [checkingTra](#)).

Usage

```
Medianwabl(F, n1 = 101, a = 1, b = 1)
```

Arguments

F	matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta, \text{inf}1, \text{sup}1, \text{sup}\theta$. The function implicitly checks if the matrix is in the correct form (tested by <code>checkingTra</code>).
n1	positive integer, by default $n1=101$. It indicates the number of desired α -levels for characterizing the φ -wabl/ldev/rdev median.
a	number >0 , by default $a=1$. It is the first parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$.
b	number >0 , by default $b=1$. It is the second parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$.

Details

See examples

Value

The function returns the φ -wabl/ldev/rdev median, given by an array of dimension $n1 \times 3 \times 1$ where $n1$ is the number of considered α -levels and 3 the number of columns of the array: the first column will be the α -levels, the second one their infimum values and the third one their supremum values.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

References

- [1] Sinova, B.; de la Rosa de Saa, S.; Gil, M.A.: *A generalized L1-type metric between fuzzy numbers for an approach to central tendency of fuzzy data*, Information Sciences 242, pp. 22-34 (2013)
- [2] Sinova, B.; Gil, M.A.; Van Aelst, S.: *M-estimates of location for the robust central tendency of fuzzy data*, IEEE Transactions on Fuzzy Systems 24(4), pp. 945-956 (2016)

See Also

[checkingTra](#), [DwablphiTra](#), [Dwablphi](#), [Wablphi](#), [Median1norm](#)

Examples

```
# Example 1:
F=SimulCASE1(10)
Medianwabl(F,3)

# Example 2:
```

```
F=matrix(c(1,0,2,3),nrow=1)
Medianwabl(F)
```

Qn

*Qn scale measure of a trapezoidal fuzzy sample***Description**

This function calculates the scale measure Qn for a matrix of trapezoidal fuzzy numbers F. The employed metric in the calculation can be the 1-norm distance, the mid/spr distance or the (φ, θ) -wabl/ldev/rdev distance. The function first checks if the input matrix F is given in the correct form (tested by checkingTra).

Usage

```
Qn(F, type, a = 1, b = 1, theta = 1/3)
```

Arguments

F	matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta, \text{inf}1, \text{sup}1, \text{sup}\theta$. The function implicitly checks if the matrix is in the correct form (tested by checkingTra).
type	number 1, 2 or 3: if <code>type==1</code> , the 1-norm distance will be considered in the calculation of the measure ADD. If <code>type==2</code> , the mid/spr distance will be considered. By contrast, if <code>type==3</code> , the (φ, θ) -wabl/ldev/rdev distance will be used.
a	number >0 , by default <code>a=1</code> . It is the first parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$ in the mid/spr distance or in the (φ, θ) -wabl/ldev/rdev distance.
b	number >0 , by default <code>b=1</code> . It is the second parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$ in the mid/spr distance or in the (φ, θ) -wabl/ldev/rdev distance.
theta	number >0 , by default <code>theta=1/3</code> . It is the weight of the spread in the mid/spr distance and the weight of the ldev and rdev in the (φ, θ) -wabl/ldev/rdev distance.

Details

See examples

Value

The function returns the scale measure Qn, which is a real number.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

See Also

[checkingTra](#), [Rho1Tra](#), [DthetaphiTra](#), [DwablphiTra](#)

Examples

```
# Example 1:
F=SimulCASE1(10)
Qn(F,3,1,1,1)

# Example 2:
F=matrix(c(1,3,2,2),nrow=1)
Qn(F,2,5,1,1)
```

Rho1

1-norm distance between fuzzy numbers

Description

This function calculates the 1-norm distance between the fuzzy numbers contained in two arrays, which should be given in the desired format. For this, the function first checks if the input arrays R and S are in the correct form (tested by `checking`) and if the α -levels of all fuzzy numbers coincide.

Usage

Rho1(R, S)

Arguments

R	array of dimension $n1 \times 3 \times r$ containing r fuzzy numbers characterized by means of $n1$ α -levels each. The function first calls <code>checking</code> to check if the array R has the correct format. Moreover, the α -levels of the array R should coincide with the ones of the array S (the function checks this condition).
S	array of dimension $n1 \times 3 \times s$ containing s fuzzy numbers characterized by means of $n1$ α -levels each. The function first calls <code>checking</code> to check if the array S has the correct format. Moreover, the α -levels of the array S should coincide with the ones of the array R (the function checks this condition).

Details

See examples

Value

The function returns a matrix of dimension $r \times s$ containing the 1-norm distances between the fuzzy numbers of the array R and the fuzzy numbers of the array S .

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

References

[1] Diamond, P.; Kloeden, P.: *Metric spaces of fuzzy sets*, Fuzzy Sets Syst. 35, pp. 241-249 (1990)

See Also

[checking](#), [Rho1Tra](#)

Examples

```
# Example 1:
F=SimulCASE1(4)
S=SimulCASE1(5)
F=TransfTra(F)
S=TransfTra(S)
Rho1(F,S)

# Example 2:
F=SimulCASE1(4)
S=SimulCASE1(5)
S=TransfTra(S)
Rho1(F,S)

# Example 3:
F=SimulCASE1(4)
S=SimulCASE1(5)
F=TransfTra(F)
S=TransfTra(S,10)
Rho1(F,S)
```

Rho1Tra

1-norm distance between trapezoidal fuzzy numbers

Description

This function calculates the 1-norm distance between the trapezoidal fuzzy numbers contained in two matrixes, which should be given in the desired format. For this, the function first checks if the input matrixes R and S are in the correct form (tested by `checkingTra`).

Usage

Rho1Tra(R, S)

Arguments

- R matrix of dimension $r \times 4$ containing r trapezoidal fuzzy numbers characterized by their four values $\text{inf}0, \text{inf}1, \text{sup}1, \text{sup}0$. The function first calls `checkingTra` to check if the matrix R has the correct format.
- S matrix of dimension $s \times 4$ containing s trapezoidal fuzzy numbers characterized by their four values $\text{inf}0, \text{inf}1, \text{sup}1, \text{sup}0$. The function first calls `checkingTra` to check if the matrix S has the correct format.

Details

See examples

Value

The function returns a matrix of dimension $r \times s$ containing the 1-norm distances between the trapezoidal fuzzy numbers of the matrix R and the trapezoidal fuzzy numbers of the matrix S.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

See Also

[checkingTra](#), [Rho1](#)

Examples

```
# Example 1:
F=SimulCASE2(4)
S=SimulCASE3(5)
Rho1Tra(F,S)

# Example 2:
F=matrix(c(1,1,0,2,3,4,5,6),nrow=2)
S=SimulCASE3(5)
Rho1Tra(F,S)
```

S1

S1 dataset

Description

S1 is a matrix of dimension 69 x 4 containing 69 trapezoidal fuzzy rating responses, each of which is characterized by its four values $\text{inf}\theta$, $\text{inf}1$, $\text{sup}1$, $\text{sup}\theta$. The data correspond to the well-known questionnaire TIMSS-PIRLS2011. This questionnaire was adapted to allow a double-type response, namely, the original Likert and a fuzzy rating scale-based (to simplify, trapezoidal). The questionnaire was conducted on 69 fourth grade students from Colegio San Ignacio (Oviedo-Asturias, Spain). Trapezoidal fuzzy rating responses to the Question S1 "My teacher taught me to discover science in daily life" are collected in this dataset.

Usage

```
data("S1")
```

Format

A matrix of dimension 69 x 4 containing 69 trapezoidal fuzzy rating responses, each of which is characterized by its four values $\text{inf}\theta$, $\text{inf}1$, $\text{sup}1$, $\text{sup}\theta$.

Details

See examples

Source

The complete dataset can be found in <http://bellman.ciencias.uniovi.es/SMIRE/FuzzyRatingScaleQuestionnaire-SanIgnacio.html>

References

- [1] Gil, M.A.; Lubiano, M.A.; De la Rosa de Saa, S.; Sinova, B.: *Analyzing data from a fuzzy rating scale-based questionnaire. A case study*, *Psicothema* 27(2), pp. 182-191 (2015)
- [2] Lubiano, M.A.; De la Rosa de Saa, S.; Montenegro, M.; Sinova, B.; Gil, M.A.: *Descriptive analysis of responses to items in questionnaires. Why not a fuzzy rating scale?*, *Information Sciences* 360, pp. 131-148 (2016)
- [3] Lubiano, M.A.; Montenegro, M.; Sinova, B.; De la Rosa de Saa, S.; Gil, M.A.: *Hypothesis testing for means in connection with fuzzy rating scale-based data: algorithms and applications*, *European Journal of Operational Research* 251, pp. 918-929 (2016)

Examples

```
data(S1)
filterNA(S1)
F=filterNA(S1)[[1]]
Var(F)
```

`SimulCASE1`*Simulation of trapezoidal fuzzy numbers CASE 1*

Description

This function generates n trapezoidal fuzzy numbers from a symmetric distribution and with independent components (for a detailed explanation of the simulation see the paper [1] below, namely, the Case 1 for noncontaminated samples).

Usage`SimulCASE1(n)`**Arguments**

`n` positive integer. It is the number of trapezoidal fuzzy numbers to be generated.

Details

See examples

Value

This function returns n trapezoidal fuzzy numbers contained in a matrix of dimension $n \times 4$. Each trapezoidal fuzzy number is characterized by its four values $\text{inf}0, \text{inf}1, \text{sup}1, \text{sup}0$.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

References

[1] Sinova, B.; Gil, M.A.; Colubi, A.; Van Aelst, S.: *The median of a random fuzzy number. The 1-norm distance approach*, Fuzzy Sets Syst. 200, pp. 99-115 (2012)

See Also

[SimulCASE2](#), [SimulCASE3](#), [SimulCASE4](#), [SimulFRSTra](#)

Examples

```
# Example 1:  
SimulCASE1(10)
```

`SimulCASE2`*Simulation of trapezoidal fuzzy numbers CASE 2*

Description

This function generates n trapezoidal fuzzy numbers from a symmetric distribution and with dependent components (for a detailed explanation of the simulation see the paper [1] below, namely, the Case 2 for noncontaminated samples).

Usage`SimulCASE2(n)`**Arguments**

`n` positive integer. It is the number of trapezoidal fuzzy numbers to be generated.

Details

See examples

Value

This function returns n trapezoidal fuzzy numbers contained in a matrix of dimension $n \times 4$. Each trapezoidal fuzzy number is characterized by its four values $\text{inf}\emptyset, \text{inf}1, \text{sup}1, \text{sup}\emptyset$.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

References

[1] Sinova, B.; Gil, M.A.; Colubi, A.; Van Aelst, S.: *The median of a random fuzzy number. The 1-norm distance approach*, Fuzzy Sets Syst. 200, pp. 99-115 (2012)

See Also

[SimulCASE1](#), [SimulCASE3](#), [SimulCASE4](#), [SimulFRSTra](#)

Examples

```
# Example 1:  
SimulCASE2(10)
```

`SimulCASE3`*Simulation of trapezoidal fuzzy numbers CASE 3*

Description

This function generates n trapezoidal fuzzy numbers from an asymmetric distribution and with independent components (for a detailed explanation of the simulation see the paper [1] below, namely, the Case 3 for noncontaminated samples).

Usage`SimulCASE3(n)`**Arguments**

n positive integer. It is the number of trapezoidal fuzzy numbers to be generated.

Details

See examples

Value

This function returns n trapezoidal fuzzy numbers contained in a matrix of dimension $n \times 4$. Each trapezoidal fuzzy number is characterized by its four values $inf0, inf1, sup1, sup0$.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

References

[1] Sinova, B.; Gil, M.A.; Van Aelst, S.: *M-estimates of location for the robust central tendency of fuzzy data*, IEEE Transactions on Fuzzy Systems 24(4), pp. 945-956 (2016)

See Also

[SimulCASE1](#), [SimulCASE2](#), [SimulCASE4](#), [SimulFRSTra](#)

Examples

```
# Example 1:  
SimulCASE3(10)
```

`SimulCASE4`*Simulation of trapezoidal fuzzy numbers CASE 4*

Description

This function generates n trapezoidal fuzzy numbers from an asymmetric distribution and with dependent components (for a detailed explanation of the simulation see the paper [1] below, namely, the Case 4 for noncontaminated samples).

Usage`SimulCASE4(n)`**Arguments**

`n` positive integer. It is the number of trapezoidal fuzzy numbers to be generated.

Details

See examples

Value

This function returns n trapezoidal fuzzy numbers contained in a matrix of dimension $n \times 4$. Each trapezoidal fuzzy number is characterized by its four values $\text{inf}0, \text{inf}1, \text{sup}1, \text{sup}0$.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

References

[1] Sinova, B.; Gil, M.A.; Van Aelst, S.: *M-estimates of location for the robust central tendency of fuzzy data*, IEEE Transactions on Fuzzy Systems 24(4), pp. 945-956 (2016)

See Also

[SimulCASE1](#), [SimulCASE2](#), [SimulCASE3](#), [SimulFRSTra](#)

Examples

```
# Example 1:  
SimulCASE4(10)
```

 SimulFRSTra

Simulation of trapezoidal fuzzy rating responses to a questionnaire

Description

This function generates n trapezoidal responses based on the fuzzy rating scale. They are simulated mimicking the human behavior, considering for it a finite mixture of three different procedures (for a detailed explanation of the simulation see the paper [1] below), and generated in the interval $[1,k]$, being k the number of Likert responses of the supposed questionnaire.

Usage

```
SimulFRSTra(n, k, w1, w2, w3, p, q)
```

Arguments

n	positive integer. It is the number of trapezoidal fuzzy numbers to be generated.
k	positive integer and >1 . It's the number of Likert responses of the supposed questionnaire. The trapezoidal fuzzy responses will be generated in the interval $[1,k]$.
$w1$	number in $[0,1]$. It should be fulfilled that $w1+w2+w3=1$.
$w2$	number in $[0,1]$. It should be fulfilled that $w1+w2+w3=1$.
$w3$	number in $[0,1]$. It should be fulfilled that $w1+w2+w3=1$.
p	number >0 . It is the first parameter of the beta distribution.
q	number >0 . It is the second parameter of the beta distribution.

Details

See examples

Value

This function returns n trapezoidal fuzzy rating responses contained in a matrix of dimension $n \times 4$, with values in the interval $[1,k]$. Each trapezoidal fuzzy rating response is characterized by its four values $\text{inf}\theta, \text{inf}1, \text{sup}1, \text{sup}\theta$.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

References

[1] De la Rosa de Saa, S.; Gil, M.A.; Gonzalez-Rodriguez, G.; Lopez, M.T.; Lubiano M.A.: *Fuzzy rating scale-based questionnaires and their statistical analysis*, IEEE Trans. Fuzzy Syst. 23(1), pp. 111-126 (2015)

See Also

[SimulCASE1](#), [SimulCASE2](#), [SimulCASE3](#), [SimulCASE4](#)

Examples

```
# Example 1:
SimulFRSTra(100,6,0.05,0.35,0.6,2,1)
```

Sn	<i>Sn scale measure of a trapezoidal fuzzy sample</i>
----	---

Description

This function calculates the scale measure Sn for a matrix of trapezoidal fuzzy numbers F. The employed metric in the calculation can be the 1-norm distance, the mid/spr distance or the (φ, θ) -wabl/ldev/rdev distance. The function first checks if the input matrix F is given in the correct form (tested by checkingTra).

Usage

```
Sn(F, type, a = 1, b = 1, theta = 1/3)
```

Arguments

F	matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta, \text{inf}1, \text{sup}1, \text{sup}\theta$. The function implicitly checks if the matrix is in the correct form (tested by checkingTra).
type	number 1, 2 or 3: if <code>type==1</code> , the 1-norm distance will be considered in the calculation of the measure ADD. If <code>type==2</code> , the mid/spr distance will be considered. By contrast, if <code>type==3</code> , the (φ, θ) -wabl/ldev/rdev distance will be used.
a	number >0 , by default <code>a=1</code> . It is the first parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$ in the mid/spr distance or in the (φ, θ) -wabl/ldev/rdev distance.
b	number >0 , by default <code>b=1</code> . It is the second parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$ in the mid/spr distance or in the (φ, θ) -wabl/ldev/rdev distance.
theta	number >0 , by default <code>theta=1/3</code> . It is the weight of the spread in the mid/spr distance and the weight of the ldev and rdev in the (φ, θ) -wabl/ldev/rdev distance.

Details

See examples

Value

The function returns the scale measure S_n , which is a real number.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

See Also

[checkingTra](#), [Rho1Tra](#), [DthetaphiTra](#), [DwablphiTra](#)

Examples

```
# Example 1:
F=SimulCASE1(10)
Sn(F,2,5,1,0.5)

# Example 2:
F=matrix(c(1,3,2,2),nrow=1)
Sn(F,1)
```

Tn

Tn scale measure of a trapezoidal fuzzy sample

Description

This function calculates the scale measure T_n for a matrix of trapezoidal fuzzy numbers F . The employed metric in the calculation can be the 1-norm distance, the mid/spr distance or the (φ, θ) -wabl/ldev/rdev distance. The function first checks if the input matrix F is given in the correct form (tested by `checkingTra`).

Usage

```
Tn(F, type, a = 1, b = 1, theta = 1/3)
```


Arguments

F	matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta, \text{inf}1, \text{sup}1, \text{sup}\theta$. The function implicitly checks if the matrix is in the correct form (tested by <code>checkingTra</code>).
type	number 1, 2 or 3: if <code>type==1</code> , the 1-norm distance will be considered in the calculation of the measure ADD. If <code>type==2</code> , the mid/spr distance will be considered. By contrast, if <code>type==3</code> , the (φ, θ) -wabl/ldev/rdev distance will be used.
a	number >0 , by default <code>a=1</code> . It is the first parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$ in the mid/spr distance or in the (φ, θ) -wabl/ldev/rdev distance.
b	number >0 , by default <code>b=1</code> . It is the second parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$ in the mid/spr distance or in the (φ, θ) -wabl/ldev/rdev distance.
theta	number >0 , by default <code>theta=1/3</code> . It is the weight of the spread in the mid/spr distance and the weight of the ldev and rdev in the (φ, θ) -wabl/ldev/rdev distance.

Details

See examples

Value

The function returns the scale measure Tn, which is a real number.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

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See Also

[checkingTra](#), [Rho1Tra](#), [DthetaphiTra](#), [DwablphiTra](#)

Examples

```
# Example 1:
F=SimulCASE1(10)
Tn(F,1)

# Example 2:
F=matrix(c(1,2,3,4),nrow=2)
Tn(F,2,5,1,0.5)
```

TransfTra

Transformation of a matrix of trapezoidal fuzzy numbers into an array

Description

This function transforms a matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta, \text{inf}1, \text{sup}1, \text{sup}\theta$ into an array of dimension $n1 \times 3 \times n$ containing these n fuzzy numbers characterized by means of $n1$ equidistant α -levels each (by default $n1=101$). The function first checks if the input matrix F is given in the correct form (tested by `checkingTra`). In case yes, the function returns an array given in the format explained in the function checking.

Usage

```
TransfTra(F, n1 = 101)
```

Arguments

F	matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta, \text{inf}1, \text{sup}1, \text{sup}\theta$. The function implicitly checks if the matrix is in the correct form (tested by <code>checkingTra</code>).
$n1$	positive integer, by default $n1=101$. It indicates the number of desired α -levels for characterizing the trapezoidal fuzzy numbers.

Details

See examples

Value

The function returns an array of dimension $n1 \times 3 \times n$ containing the n trapezoidal fuzzy numbers characterized by means of $n1$ α -levels. The first column of the array are the α -levels, the second one their infimum values and the third one their supremum values. The correct format of the array is explained in the function checking.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

See Also

[checkingTra](#), [checking](#)

Examples

```
# Example 1:
F=SimulCASE3(10)
TransfTra(F,200)

# Example 2:
F=matrix(c(1,1,0,2,3,4,5,6),nrow=2)
TransfTra(F)
```

Var *Variance of a trapezoidal fuzzy sample*

Description

Given a sample of trapezoidal fuzzy numbers contained in a matrix F, the function calculates the variance of these numbers with respect to the mid/spr distance. The function first checks if the input matrix F is given in the correct form (tested by checkingTra).

Usage

```
Var(F, a = 1, b = 1, theta = 1/3)
```

Arguments

F	matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta, \text{inf}1, \text{sup}1, \text{sup}\theta$. The function implicitly checks if the matrix is in the correct form (tested by checkingTra).
a	number >0 , by default $a=1$. It is the first parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$.
b	number >0 , by default $b=1$. It is the second parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$.
theta	number >0 , by default $\text{theta}=1/3$. It is the weight of the spread in the mid/spr distance.

Details

See examples

Value

The function returns the variance of the sample with respect to the mid/spr distance, which is a real number.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

References

[1] De la Rosa de Saa, S.; Lubiano M.A.; Sinova, B.; Filzmoser, P.: *Robust scale estimators for fuzzy data*, Advances in Data Analysis and Classification, pp. 1-28 (2015)

See Also

[checkingTra](#), [Mean](#), [DthetaphiTra](#)

Examples

```
# Example 1:
F=SimulCASE1(10)
Var(F,1,1,1)

# Example 2:
F=matrix(c(1,0,2,3),nrow=1)
Var(F)
```

Wablphi

φ -wabl values of a trapezoidal fuzzy sample

Description

Given a sample of trapezoidal fuzzy numbers contained in a matrix F, the function calculates the φ -wabl value for each of these numbers. The function first checks if the input matrix F is given in the correct form (tested by `checkingTra`).

Usage

```
Wablphi(F, a = 1, b = 1)
```

Arguments

F	matrix of dimension $n \times 4$ containing n trapezoidal fuzzy numbers characterized by their four values $\text{inf}\theta, \text{inf}1, \text{sup}1, \text{sup}\theta$. The function implicitly checks if the matrix is in the correct form (tested by <code>checkingTra</code>).
a	number >0 , by default $a=1$. It is the first parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$.
b	number >0 , by default $b=1$. It is the second parameter of a beta distribution which corresponds to a weighting measure on $[0,1]$.

Details

See examples

Value

The function returns a vector giving the φ -wabl values of each trapezoidal fuzzy number.

Note

In case you find (almost surely existing) bugs or have recommendations for improving the functions comments are welcome to the above mentioned mail addresses.

Author(s)

Asun Lubiano <lubiano@uniovi.es>, Sara de la Rosa de Saa <rosasara@uniovi.es>

References

[1] Sinova, B.; Gil, M.A.; Lopez, M.T.; Van Aelst, S.: *A parameterized L2 metric between fuzzy numbers and its parameter interpretation*, Fuzzy Sets and Systems 245, pp. 101-115 (2014)

See Also

[checkingTra](#), [DwablphiTra](#), [Dwablphi](#), [Medianwabl](#)

Examples

```
# Example 1:  
F=SimulCASE4(60)  
Wablphi(F,2,1)  
  
# Example 2:  
F=matrix(c(1,0,2,3),nrow=1)  
Wablphi(F)
```

Index

- * **NA**
 - filterNA, 15
- * **arith**
 - Dthetaphi, 9
 - DthetaphiTra, 10
 - Dwablphi, 12
 - DwablphiTra, 13
 - Rho1, 30
 - Rho1Tra, 31
- * **attribute**
 - checking, 6
 - checkingTra, 7
- * **datagen**
 - SimulCASE1, 34
 - SimulCASE2, 35
 - SimulCASE3, 36
 - SimulCASE4, 37
 - SimulFRSTra, 38
- * **datasets**
 - M1, 20
 - M2, 21
 - M3, 22
 - S1, 33
- * **manip**
 - TransfTra, 42
- * **package**
 - FuzzyStatTra-package, 2
- * **robust**
 - M.estimate, 18
 - MDD, 23
 - Median1norm, 26
 - Medianwabl, 27
 - Qn, 29
 - Sn, 39
 - Tn, 40
- * **survey**
 - M1, 20
 - M2, 21
 - M3, 22
- S1, 33
- * **univar**
 - ADD, 4
 - GSI, 16
 - HyperI, 17
 - M.estimate, 18
 - MDD, 23
 - Mean, 25
 - Median1norm, 26
 - Medianwabl, 27
 - Qn, 29
 - Sn, 39
 - Tn, 40
 - Var, 43
 - Wablphi, 44
- ADD, 4
- checking, 5, 6, 8, 10, 13, 15, 24, 31, 42
- checkingTra, 5, 6, 7, 11, 14, 15, 17–19, 24, 26–28, 30, 32, 40–42, 44, 45
- Dthetaphi, 5, 9, 11, 24
- DthetaphiTra, 5, 10, 10, 19, 24, 30, 40, 41, 44
- Dwablphi, 5, 12, 14, 24, 28, 45
- DwablphiTra, 5, 13, 13, 19, 24, 28, 30, 40, 41, 45
- filterNA, 6, 8, 15
- FuzzyStatTra (FuzzyStatTra-package), 2
- FuzzyStatTra-package, 2
- GSI, 16
- HyperI, 17
- M.estimate, 18
- M1, 20
- M2, 21
- M3, 22
- MDD, 23

Mean, [25](#), [44](#)

MedianInorm, [26](#), [28](#)

Medianwabl, [27](#), [27](#), [45](#)

Qn, [29](#)

Rho1, [5](#), [24](#), [30](#), [32](#)

Rho1Tra, [5](#), [19](#), [24](#), [30](#), [31](#), [31](#), [40](#), [41](#)

S1, [33](#)

SimulCASE1, [34](#), [35–37](#), [39](#)

SimulCASE2, [34](#), [35](#), [36](#), [37](#), [39](#)

SimulCASE3, [34](#), [35](#), [36](#), [37](#), [39](#)

SimulCASE4, [34–36](#), [37](#), [39](#)

SimulFRSTra, [34–37](#), [38](#)

Sn, [39](#)

Tn, [40](#)

TransfTra, [5](#), [24](#), [27](#), [42](#)

Var, [43](#)

Wablphi, [13](#), [14](#), [28](#), [44](#)